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QMF FILTER DESIGN USING FOURIER AND WAVELET TRANSFORM

Amrita Khera[1], Mr.Sachin bandewaer[2], Mr. Anand kumar singh[3]

SSSCE (M.P. Bhopal), India[1]

(SSSCE, MP. Bhopal) India [2]

RKDF, University (MP. Bhopal) India [3]

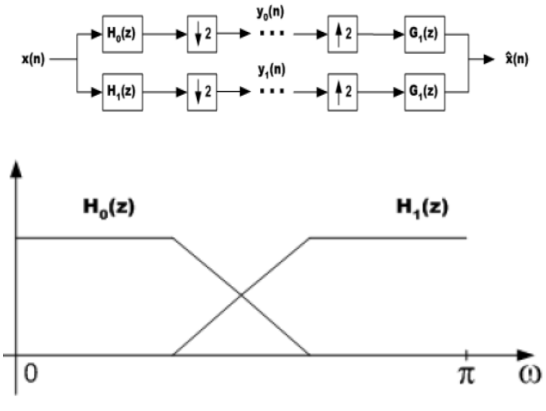
Abstract: The paper represents the theory of designing of quadrature mirror filter (QMF).it was resent that Mc Clellen transform could be used to generate 2- d diamond shape QMF Filter. In this paper the problem of identifying frequencies of disturbances in flexible systems using advanced Digital Signal processing techniques such as filter banks and Quadrature Mirror Filters is addressed. In a number of situations there is a need to design a controller for a system with flexible modes In this paper the problem of identifying frequencies of disturbances in flexible systems using advanced Digital Signal Processing techniques such as filter banks and Quadrature Mirror Filters is addressed. In a number of situations there is a need to design a controller for a system with flexible modes This includes design of decimation and interpolation filters, analysis/synthesis filter banks (also called quadrature mirror filters, or QMf.

I. INTRODUCTION

This paper presents the theory of the wavelet transform (WT) and its connection to the theory of multirate filter banks. The wavelet transform was first introduced in the mathematical literature by Grossmann and Morlet in 1984, and further treated by Meyer, Daubechies, Mallat, and others in the late 1980's.In particular, works by Daubechies and Mallat established the connection between wavelets and digital filter banks that, as a result, generated much interest and activity in the respective areas. The theory of multirate filter banks, on the other hand, was first developed in the context of coding applications in the late 1970's by Croisier, Esteban, and Galand who introduced a special class of filters called quadrature mirror filters (QMF), and also by rochiere, Webber, and Flanagan who introduced a similar technique in the context of speech coding . Subsequently, solutions to the perfect reconstruction (PR) filter bank for the two-band and the general M-band case were found, and a general theory on the design of multirate filter banks was also established. Some historical perspectives on the development of wavelets and filter banks can be found in, and in-depth studies of wavelets and filter banks can be found in. This paper is organized as describes maximally decimated two channel filter banks, presents the wavelet transform in the continuous-time and discrete-time domain and shows its relationship to the two-channel filter bank, covers design issues of the wavelet filter bank, and ends with a brief summary.

II. THE TWO-CHANNEL FILTER BANK

Digital Filter banks are commonly used in applications that require a way of transforming the input signal into a frequency or time-frequency domain representation. As the name suggests, this is done through a bank of filters that divides the signal spectrum into approximate frequency subbands or channels and generates a time-indexed series of coefficients that represent the frequency-localized signal energy within each band A uniform two-channel filter bank is shown in Figure 4.1(a) and the corresponding magnitude response in Figure 4.1b).



(b)

Figure 4.1: Two-channel filter bank (a) analysis and synthesis filter bank structure (b) frequency response of analysis filters $H0(z)$ and $H1(z)$

In the analysis stage, the input signal $x(n)$ is filtered by the low-pass filter $H0(z)$ and the high-pass filter $H1(z)$ and then down-sampled by a factor of 2 to produce subband signals $y0(n)$ and $y1(n)$, respectively. In the synthesis stage, the subband signals $y0(n)$ and $y1(n)$ are first up-sampled by a factor of 2, then passed through low-pass filter $G0(z)$ and high-pass filter $G1(z)$, respectively, and finally added together to produce the reconstructed signal $\hat{}x(n)$. In the z-domain, the down-sampling and up-sampling operations can be expressed as

$$g(n) = (\downarrow 2)f(n) : G(z) = 1\backslash 2[F(z^{1/2}) + F(-z^{1/2})] \quad (4.1)$$

$$g(n) = (\uparrow 2)f(n) : G(z) = F(z^2). \quad (4.2)$$

Using equations 4.1 and 4.2 and the input-output relationship of the filter bank in Figure4.1, we obtain

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$$\hat{X}(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\}X(z) + \frac{1}{2}\{H_0(-z)G_0(z) + H_1(-z)G_1(z)\}X(-z) \quad (4.3)$$

where the first term represents the amplitude and phase distortions that result from thefiltering operations and the second term represents the aliasing and imaging distortionsthat result from the down-sampling and up-sampling operations. The first term is calledthe *distortion transfer function*, $T(z)$, and the second term is called the *aliasing transferfunction*

$$T(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\}, \quad (4.4)$$

$$A(z) = \frac{1}{2}\{H_0(-z)G_0(z) + H_1(-z)G_1(z)\}. \quad (4.5)$$

Since any distortion caused by the filter bank is undesirable, especially aliasing error [37],the design of the analysis and synthesis filters revolve around the requirements of *aliascancellation* (AC) and *perfect reconstruction* (PR). The conditions for AC and PR can be summarized as follows.

- Alias Cancellation: Choose the synthesis filters as

$$G_0(z) = H_1(-z), \quad (4.6)$$
$$G_1(z) = -H_0(-z). \quad (4.7)$$

Then,

$$A(z) = \frac{1}{2}\{H_0(-z)H_1(-z) - H_1(-z)H_0(-z)\} = 0. \quad (4.8)$$

Notice that the AC condition simplifies the design to designing only filters $H0(z)$ and $H1(z)$ and minimizing the distortion in $T(z)$.

- Perfect Reconstruction: For PR, we need

$$T(z) = cz^{-l}, \quad (4.9)$$

where c = constant and $l \in$ Z, and

$$A(z) = 0,$$

so that (with $c = 1$)

$$\hat{X}(z) = T(z)X(z) + A(z)X(-z) = z^{-l}X(z) + (0)X(-z) = z^{-l}X(z), \quad (4.11)$$

where the reconstructed signal is just a delay of the input signal by $z-l$.

III. CLASSIC QMF FILTERS (NON-PR)

The "classic" QMF filters proposed by Croisier, Esteban, and Galand [41] are designed by first imposing the relationship

$$H_1(z) = H_0(-z) \quad \text{or} \quad h_1(n) = (-1)^n h_0(n), \quad (4.12)$$

which relates the low-pass and high-pass filter through a simple sign alteration. Equation
4.12 can also be expressed in the Fourier domain as

$$|H_1(e^{jw})| = |H_0(e^{j(\pi-w)})|. \quad (4.13)$$

$H1(ejw)$ in equation 4.13 represents a high-pass filter whose response is a mirror image of the low-pass filter response $|H0(ejw)|$ with respect to the quadrature frequency, π 2 . Using the AC condition of equations 4.6 and 4.7, and equation 4.12 above, the distortion transfer function can now be simplified to

$$T(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\} = \frac{1}{2}\{H_0(z)H_1(-z) - H_0(-z)H_1(z)\} = \frac{1}{2}\{H_0^2(z) - H_0^2(-z)\}. \quad (4.14)$$

Note that the design of QMF filters according to 4.14 only involves one filter, $H0(z)$. Several well known solutions to this exist and a few are described next. First, note that for PR we need

$$T(z) = \frac{1}{2}\{H_0^2(z) - H_0^2(-z)\} = z^{-l}. \quad (4.15)$$

The only solution to 4.15 using an FIR filter is the trivial Haar filter as all other solutions involve some type of distortion in $T(z)$. Among more practical FIR solutions, Johnston's

Filters offer small reconstruction error and good overall performance. Johnston's filters are designed to provide high stop-band attenuations and good transition-band characteristics while eliminating phase distortion and minimizing amplitude distortion in $T(z)$. Among IIR solutions, the well known elliptic filters offer a solution where amplitude distortion is eliminated and phase distortion is minimized.

IV. SMITH-BARNWELL FILTERS (PR ORTHOGONAL)

The solution proposed by Smith and Barnwell [66] is based on the AC condition and the Relationship

$$H_1(z) = -z^{-N}H_0(-z^{-1}), \quad (4.16)$$

where filters $H0(z)$ and $H1(z)$ (as well as $G0(z)$ and $G1(z)$) are FIR filters of odd order N. Also called *conjugate quadrature filters* (CQF), these filters provide the quadrature mirror property like QMF filters, but also the perfect reconstruction property as $T(z)$ can now be made to be a pure delay. The distortion function $T(z)$ can be simplified using equations 4.6, 4.7, 4.16 as

$$T(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\} = \frac{1}{2}\{H_0(z)H_1(-z) - H_0(-z)H_1(z)\} = \frac{z^{-N}}{2}\{H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1})\}. \quad (4.17)$$

Note that the design of CQF filters also involves only one filter, $H0(z)$, as the other three can be derived using equations 4.6, 4.7, 4.16. To obtain PR in equation 4.17, we need

$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 2. \quad (4.18)$$

If we define

$$P(z) = H_0(z)H_0(z^{-1}), \quad (4.19)$$

then we can re-write 4.18 as

$$P(z) + P(-z) = 2. \quad (4.20)$$

$P(z)$ represents a zero-phase *half-band* filter in which all even-indexed terms are zero except the term at $z0$. Description and design of half-band filters have already been discussed extensively in the filter bank literature, e.g. [35]. Once half-band filter $P(z)$ is designed, filter $H0(z)$ can be obtained through symmetrical factorization of 4.19 [67]. In addition to PR, Smith-Barnwell filters also

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provide the orthogonality property that is described next. First, using equation 4.16 in 4.18, we obtain

$$H_1(z)H_1(z^{-1}) + H_1(-z)H_1(-z^{-1}) = 2. \tag{4.21}$$

Next, we can re-write equation 4.16 and obtain

$$H_0(z) = -z^{-N}H_1(-z^{-1}), \tag{4.22}$$

and using the equality relationship given by

$$H_0(z^{-1})H_1(z) = H_0(z^{-1})H_1(z), \tag{4.23}$$

we can substitute 4.23 in equations 4.16 and 4.22 to obtain

$$\begin{aligned} H_0(z^{-1})H_1(z) &= (-z^N H_1(-z))(z^{-N}H_0(-z^{-1})) \\ H_0(z^{-1})H_1(z) &= -H_1(-z)H_0(-z^{-1}) \\ H_0(z^{-1})H_1(z) &+ H_1(-z)H_0(-z^{-1}) = 0. \end{aligned} \tag{4.24}$$

Equations 4.18, 4.21, and 4.24 represent the orthogonality condition in the z-domain. The term $Hi(z)Hi(z-1)$ in equations 4.18 and 4.19 represents the auto-correlation of $Hi(z)$, and the term $H0(z-1)H1(z)$ in 4.24 represents the cross-correlation between $H0(z)$ and $H1(z)$ [68]. Equations 4.18 and 4.21 are also known as the power symmetric property [35]. In the time domain, equations 4.18, 4.21, and 4.24 can be expressed as

$$\sum_n h_0(n)h_0(n+2k) = \delta(k), \tag{4.25}$$

$$\sum_n h_1(n)h_1(n+2k) = \delta(k), \tag{4.26}$$

$$\sum_n h_0(n)h_1(n+2k) = 0, \tag{4.27}$$

or more succinctly as

$$\sum_n h_i(n)h_j(n+2k) = \delta(i-j)\delta(k) \tag{4.28}$$

Where

$$\begin{aligned} h &= \text{analysis filters} \\ g &= \text{synthesis filters} \\ i,j &= \text{0 for low-pass, 1 for high-pass} \\ k &\in \mathbb{Z}. \end{aligned}$$

In general, Smith-Barnwell filters provide PR, finite support, and orthogonality, but lack linear phase (except for the trivial Haar filter).

V. GENERALIZED QMF FILTERS (PR LINEAR PHASE)

Generalized QMF filters represent PR solutions that sacrifice orthogonality for linear phase. Using equation 4.4 and the AC condition, we obtain

$$\begin{aligned} T(z) &= \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\} \\ &= \frac{1}{2}\{H_0(z)H_1(-z) - H_0(-z)H_1(z)\} \end{aligned} \tag{4.29}$$

where filters $H0(z)$ and $H1(-z)$ can be of even or odd order and the lengths of the two are not necessarily equal. Unlike the CQF filters, the design now involves first designing the two analysis filters $H0(z)$ and $H1(-z)$, and then obtaining the two synthesis filters using equations 4.6 and 4.7. To satisfy PR, we impose

$$H_0(z)H_1(-z) - H_0(-z)H_1(z) = 2z^{-2l-1} \tag{4.30}$$

where $l \in$ Z. Note that the delay term on the right-hand side has to be odd since all even terms of $H0(z)H1(-z)$ cancel with the even terms of $H0(-z)H1(z)$. Defining

$$P(z) = z^{2l+1}H_0(z)H_1(z) \tag{4.31}$$

we can formulate the PR condition as

$$P(z) + P(-z) = 2, \tag{4.32}$$

which again represents a zero-phase half-band filter. However, since orthogonality is no longer required, $P(z)$ in 4.31 is no longer factored symmetrically but factored so as to provide symmetry in $H0(z)$ and $H1(z)$ separately. Detail and examples of this procedure can be found in [35, 64]. Similar to the orthogonality condition given in the z-domain and time-domain, we can summarize the biorthogonality condition in the z-domain as

$$H_0(z)G_0(z) + H_1(z)G_1(z) = 2 \tag{4.33}$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \tag{4.34}$$

and in the time-domain as

$$\sum_n h_i(n)g_j(2k-n) = \delta(i-j)\delta(k)(35?) \tag{4.35}$$

$$\begin{aligned} h &= \text{analysis filters} \\ g &= \text{synthesis filters} \\ i,j &= \text{0 for low-pass, 1 for high-pass} \\ k &\in \mathbb{Z}. \end{aligned}$$

Note that biorthogonality is a more general condition that provides orthogonality across the analysis and synthesis filters [40], as opposed to within the analysis and synthesis filters, and hence the name "bi-orthogonal".

VI. CONCLUSION

Two-channel filter banks, in general, are characterized by the type of errors they introduce into the signal and the properties that the filters provide. Reconstruction error is made up of three components, namely,) aliasing distortion, 2) amplitude distortion, and 3) phase distortion. Aliasing (and imaging) distortion is represented by $A(z)$, and amplitude and phase distortions are represented by $T(z)$. Properties of filters that we are particularly interested in are 1) finite support (i.e. FIR) 2) orthogonality, and 3) linear phase. Ideally, all three properties need to be incorporated into the filters as they are considered important in audio coding, e.g. orthogonality ensures that quantization noise in different channels remain independent, linear phase provides constant group delay, and finite support leads to stable and simple implementations. But it has been found that only two out of the three properties can be satisfied simultaneously for any given twochannel PR filter bank. This limitation is illustrated in Figure 4.2 where different solutions to the two-channel PR filter bank are shown. Regions of solutions for the three properties are shown where we find regions that offer two out of the three properties, but none that offer all three, except at the center point where the three properties overlap (i.e. Haar solution).

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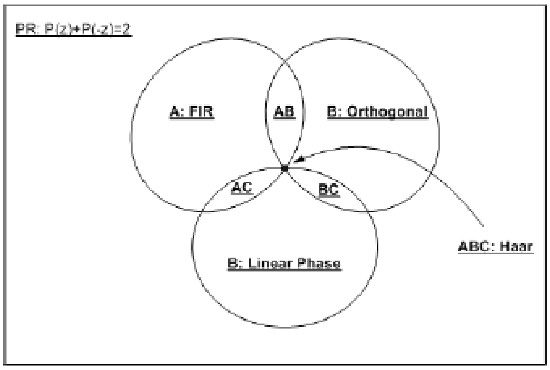

Figure 4.2: Two-channel PR filter bank solutions Venn diagram for 1) finite support, 2) orthogonality, and 3) linear phase (P(z) is rational and real)

We can summarize the two-channel filter bank solutions described in this section according to Table 4.1. Table 4.1 shows a convenient description of the four families of filters using the properties that revolve around PR. Note that, in

Filter Family	Distortions			Competing Properties		
	ALD	AD	PD	FIR	Orthogonal	Linear Phase
Johnston	None	Min.	None	Yes	?	Yes
Elliptic	None	None	Min.	No	?	No
Smith-Barnwell	None	None	None	Yes	Yes	No
Generalized QMF	None	None	None	Yes	No	Yes

Table 4.1: Two-channel filter bank solutions described in terms of properties that revolve around PR

addition to these properties, filter banks generally need to be designed to provide other important properties such as good stopband attenuation, sharp cut-off rate, low pass-band and stop-band ripples, and short delay. Filter Distortions Competing Properties Family ALD AD PD FIR Orthogonal Linear Phase Johnston None Min. None Yes ? Yes Elliptic None None Min. No ? No Smith-Barnwell None None None Yes Yes No Generalized QMF None None None Yes No Yes Table 4.1: Two-channel filter bank solutions described in terms of properties that revolve around PR It is interesting to note that during the development of two-channel (and the more general M-channel) filter banks, the so called *polyphase* representation provided a considerable amount of simplification in theory, design, and implementation. The polyphase representation is essentially a regrouping of terms in the z-domain that allows an efficient representation of the filter bank according to analysis and synthesis polyphase matrices. Some important constraints such as AC, PR, and orthogonality can be rather conveniently expressed using these matrices. As a result, much of the filter bank theory discussed today is based on the polyphase representation.

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