BINARY AND TERNARY SEQUENCE GENERATION USING IMPROVED LOGISTIC MAP

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Abstract: Pulse compression is a signal processing technique mainly used in radar, sonar and multiple access communication to achieve desired detection range and range resolution simultaneously. In this paper improved logistic map is used to generate a binary and ternary sequences with low PSR to achieve desired detection range and range resolution simultaneously. The result is analysed with each sequence and it was found that the discrimination factor for ternary sequence is higher than that of binary i.e. PSR for ternary is much lower than that of binary. This simulation results show that the ternary, a high resolution codes are good in performance, which brings our sequences closer to practical application than others.

Keywords: Pulse compression, autocorrelation, discrimination factor, PSR.

I. INTRODUCTION

Ideally a radar signal should be designed to yield high resolution, low probability of intercept and interference, optimum use of frequency spectrum, high power, low design cost and simplicity of generation. Pulse compression is a signal processing technique mainly used in radar, sonar and multiple access communication to minimize the peak transmission power, maximize SNR, and to get better resolution. This is achieved by modulating the transmitted pulse and then correlating the received signal with the transmitted pulse. The narrower the pulse width or duration the better is the range resolution. But, if the pulse width is decreased, the amount of energy in the pulse is decreased and hence maximum range detection gets reduced [1]. To overcome this problem pulse compression techniques are used in the radar systems which compresses the pulse width without reducing energy [1]. Pulse compression uses reduced transmitted power to achieve the desired range resolution. A matched filter, the front end of the receiver is used to maximize the signal to noise ratio of the received signal [4]. Different pulse compressed waveforms have been used for this purpose, including the linear frequency modulated signal, the nonlinear frequency modulated signal, biphase modulated sequences and polyphase modulated sequences. In pulse compression, the code is used to modulate the carrier waveform during transmission, which is used as the reference signal at the receiver. At the receiver front end, the reference signal is used to combine with the received signal to achieve a high range resolution. Phase coded waveforms divide the long pulse into N time segments, referred to as chips, and apply a different phase to each [4]. A very small disturbance in initial condition produces a drastically different final solution for the chaotic system and the long-term prediction is not possible. This happens even though these systems are deterministic, fully determined by meaning that their future behavior is their initial conditions, with no random elements involved in this process. This behavior is known as deterministic chaos, or simply chaos [11]. Chaotic Sequences can be generated using different types of chaotic maps. Some of the maps are Henon Map, Lorenz Map, Logistic Map, Improved logistic map, quadratic map, Tent map and Cubic map. The chaotic sequence generated by improved logistic map is neither periodic nor converging and it is sensitively dependent on initial value [4]. Earlier a binary phase code generated using chaotic map is used to achieve a low PSLR was reported [12]. In this paper similar concept has been implemented to generate ternary sequences using improved logistic map. Further their performances are compared and it was found that ternary sequence...
generated using improved logistic map gives better range resolution and detection range.

II. METHODOLOGY

A. Logistic

A great deal of chaotic behavior can be described by the one known as logistic map. Chaos is a kind of phenomena which arises in deterministic nonlinear dynamical systems which is aperiodic and sensitively depends on its initial value. The logistic map is recursive, meaning that the third term is a function of the second, the fourth a function of the third and so on. This is written as

\[ x_{n+1} = f(x_n) = \mu x_n(1 - x_n) \]  
\[ \text{-------- (1)} \]

Where \( \mu \) is some constant and is called the bifurcation parameter. Generally \( \mu \) is a number chosen between 0 and 4, and it is kept constant while the iteration process is in run. It turns out that whatever be the initial value of \( x \), after some tens or hundreds of iterations, the values of \( x \) settle into a definite pattern. This pattern can be constant, periodic or chaotic, depending on the value of \( \mu \). Values of \( \mu \) smaller than 3 give a constant value. Then starts a period doubling with a second bifurcation at 3.5, chaos shortly afterwards, and 3-step period around \( \mu = 3.8 \).

Logistic-map sequences are inherently deterministic and extremely sensitive to the initial condition \( x_0 \) and the quantization levels. The logistic map based binary and ternary Phase code (LMBPC & LMTC) derives directly from the logistic map equation. A raw sequence \( x(n) \) is first obtained by setting \( \mu = 4 \) in equation (1), ensuring the logistic map in chaotic region. The sequence is then quantized into three levels as per equation (2) to obtain the ternary sequence \( y(n) \).

\[ y(n) = \begin{cases} -1 & \text{if } x(n) < 0.3 \\ 0 & \text{if } 0.3 < x(n) < 0.7 \\ 1 & \text{if } x(n) > 0.7 \end{cases} \]  
\[ \text{-------- (2)} \]

The logistic map equation is extremely sensitive to the initial condition. By varying the initial condition \( x_0 \) and also the quantization levels a totally uncorrelated binary and ternary code sequence can be obtained. Therefore, the number of available LMBPC and LMTC waveforms at a given length is virtually infinite. The generation of LMBPC and LMTC codes is simple, fast and reproducible. A Sequence could be termed as good if it has large discrimination Factor (D). From the codes \( y(n) \) of length \( N \) obtained, good codes are selected based on the Discrimination factor which is defined as

\[ D = \frac{r(0)}{\text{Max} \ |r(k)|} \quad k \neq 0 \]  
\[ \text{-------- (3)} \]

Where \( r(k) \) is the autocorrelation of the sequence which is defined as

\[ N-1-k \]
\[ r(k) = \sum y_i y_{i+k} \]
\[ \text{Where } i = 0 \text{ to } N-1-k \text{ & k} = 0,1,2,\ldots,\ N-1 \]

and \( y(n) \) is nothing but the output of the matched filter.

Finally before finishing the discussion of pulse compression, the choice of a pulse compression system is dependent upon the type of waveform selected and the method of generation and processing. The primary factors influencing the selection of a particular waveform are radar requirements, range coverage, Doppler coverage, range and Doppler sidelobe levels, waveform flexibility, interference rejection and signal to noise ratio (SNR). The second one is the merit factor MF which is defined as the ratio of energy in the main peak of the ACF to the total energy in the sidelobes. As the signal is real valued the ACF is real and symmetric about the zero delay. For a good sequence or code the PSL should be low and MF should be high.

B. Improved Logistic Map

The improved logistic map is governed by the equation:

\[ X(n+1) = f(Xn) = 1 - 2(Xn)^2 \quad Xn \in (-1,1) \]  
\[ \text{------------------- (5)} \]

The chaotic sequence generated by improved logistic map is neither periodic nor converging and it is sensitively dependent on initial value. This equation shows chaotic behavior for an initial value of \( x_0 \) varying from 0 to 1. When \( x_0 > 1 \) as \( n \) tends to infinity, \( x_n \) also tends to infinity.

Proposed Technique

By improved logistic mapping we can generate different sequences and can select the best sequence among the sequences and thus by slightly changing initial values and bifurcation value we can generate a new different sequence. The best sequence is taken and is coded in binary and ternary for analysis. The threshold for the binary codes is given below:

\[ X (n) > 0 \text{ xx (n) = 1 } \]  
\[ X (n) < 0 \text{ xx (n) = -1 } \]

And for ternary codes

\[ X (n) = 0.7 \text{ xx (n) = 1 } \]  
\[ X (n) < -0.7 \text{ xx (n) = -1 else xx (n) = zero } \]

By applying, the Function thus been applied is auto –correlation
X (n) is an N length sequence the auto correlation function is defined as \( R (k) = \sum x(n) x(n+k) \) limits from \( n=0 \) to \( N-1 \).

At every lengths the best sequence having the highest discrimination factor are found.

**Peak Sidelobe Ratio (PSR)**

One of the most commonly used performance measures is the Peak Sidelobe Ratio. The peak sidelobe is the largest sidelobe in the correlation of a sequence. The Peak Sidelobe Ratio is defined from the autocorrelation pattern \( r(k) \) as the ratio of the maximum peak sidelobe amplitude to the mainlobe peak amplitude and is expressed in decibels.

It is given by

\[
PSR = 20 \log_{10} \max (|r(k)|) \quad \text{where} \quad k \neq 0
\]

A sequence is defined as a good sequence if the PSR of the sequence is low. The reciprocal of PSR is known as the Discrimination Factor (D.F), which is expected to have a maximum value for better range detection.

**Method of Generation of Binary and Ternary Sequences**

Ternary sequences have alphabets 0, \( \pm 1 \) where as binary sequence alphabets are \( \pm 1 \). The algorithm or method used for generating ternary sequences of length \( n \) using a chaotic map is given below:

**Step 1.** Ensure that initially the chaotic map is in the chaotic region. In logistic map the bifurcation parameter \( \mu \) is selected to be 4 so that the map is in chaotic region.

**Step 2.** Select an initial value for \( x_n \)

**Step 3.** Generate a raw sequence \( x_n \) using the chaotic map equations.

**Step 4.** The sequence is then quantized into three defined levels based on the threshold levels \( a \) and \( b \) as per equation (2) and to obtain the binary and ternary sequence \( y_n \), the threshold levels were chosen randomly between \( x_{\min} \) and \( x_{\max} \) so that a low value of PSLR is obtained. For example the values of \( x_{\min} \) and \( x_{\max} \) for logistic map are 0 and 1 respectively. The threshold levels are also chosen considering approximately the mean of the raw sequence obtained which is 0.5 in the case of logistic map.
Ternary (1,0,-1) Sequences for Length 20 & 100

L=20
0 1 -1 0 0 1 -1 -1 -1 0
0 0 1 0 1 0 1 0 0 1

L=100
0 1 -1 0 0 1 -1 -1 -1 0 0
0 0 1 0 1 0 1 0 0 1 -1
0 0 0 1 -1 0 0 0 1 0
0 1 0 1 0 1 0 1 0 1 -1
-1 0 0 1 -1 -1 -1 0 1 -1
0 0 1 -1 -1 0 0 1 0 0
0 1 0 1 -1 -1 -1 -1 0 1
0 1 -1 0 0 1 -1 0 1 -1 0 0
1 0 1 -1 0 0 0 1 -1 0 0 0

Fig.1 Auto-correlation pattern of Binary Sequence of length 20.

Fig.2 Auto-correlation pattern of Binary Sequence of length 100.

Fig.3 Auto-correlation pattern of Binary Sequence of Length 3000.

Fig1. Auto-correlation pattern of ternary Sequence of length 20.
Fig. 2: Auto-correlation pattern of Ternary Sequence of length 100

Fig. 3: Auto-correlation pattern of Ternary Sequence of Length 3000.

TABLE

<table>
<thead>
<tr>
<th>S/N</th>
<th>Length of the sequence</th>
<th>Discrimination Factor (Binary Codes)</th>
<th>Discrimination Factor (Ternary Codes)</th>
</tr>
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<tr>
<td>1</td>
<td>20</td>
<td>7.0000</td>
<td>9.0000</td>
</tr>
<tr>
<td>2</td>
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<td>7.5000</td>
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<td>40</td>
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<td>7.2857</td>
<td>7.0000</td>
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<td>7</td>
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</table>

Table on discrimination factor of binary and ternary sequences at 100 iteration, $\mu=4$.

IV. CONCLUSION

Binary and Ternary sequences were generated using improved logistic map equation. At different lengths, good sequences were obtained and it was found that the discrimination factor increases with the length of the sequence for binary and ternary codes. Better sequences are found using ternary codes because its discrimination factor is slightly greater than that of binary. These sequences are able to achieve desired detection range and range resolution simultaneously.

REFERENCES