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Construction of 16-QAM OFDM Codes with Reduced Peak to Average Power Ratio using Golay Complementary Sequences

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Abstract—OFDM is a powerful multicarrier transmission technique used extensively for wireless applications. High PAPR is one of the deleterious problems of OFDM. This paper reviews basic OFDM system and PAPR problem associated with it. Also, the construction of M-QAM particularly 16-QAM sequences using QPSK Golay sequences over $\mathbb{Z}_4$ is conferred. It is elucidated that a 16-QAM constellation can be written as the vector sum of two QPSK constellations respectively. The Golay complementary sequences over $\mathbb{Z}_4$ are then used to construct 16-QAM OFDM sequences with low PAPR. The PAPR bound for the generated 16-QAM sequences is calculated to be 5.56dB.

Keywords—OFDM, PAPR, M-QAM, QPSK, Golay Complementary Sequences.

I. INTRODUCTION

OFDM (Orthogonal frequency division multiplexing) is a special subset of Multicarrier communications, which is based on the principle of transmitting simultaneously many narrow- band orthogonal frequencies, often called OFDM subcarriers or subcarriers. OFDM has emerged as the standard of choice in a number of important high data applications due to its inherent robustness against multipath fading channels, easy implementation and high spectral efficiency [1]. OFDM-based systems are more immune to impulse noise, fast fades and can be of greater pursuit for wireless applications as they eliminate the need for highly sophisticated equalizers. Also, efficient hardware implementations can be realized using Fast Fourier Transform (FFT) techniques for small numbers of carriers. Subcarriers are made orthogonal to each other which eliminate the interference between channels and use the spectrum much more efficiently by spacing the channels more closely together. OFDM achieves robustness against multipath fading because of longer symbol duration for each subcarrier and against the intersymbol interference by inserting a guard interval in every OFDM symbol.

High Peak to Average Power Ratio is one of the major practical complications of the uncoded OFDM signals. A higher PAPR means the system requires large linearity range for the analog circuitry used and increase in the dynamic range of the analog-to-digital converter. This causes expensive system elements, higher power consumption and lower efficiency of the power amplifier (PA).

PAPR reduction methods can be categorized as distortion type e.g. clipping, peak windowing, peak cancellation etc and non-distortion type methods e.g. peak reduction carrier, selective level mapping (SLM), partial transmit sequence (PTS) and block coding [2]. Coding method not only reduces PAPR to greater extent but also has the advantage of error correction associated with it. Golay codes are an excellent example of coding method whose PAPR is at most 2. Davis and Jedwab disclosed a relationship between Golay Complementary Sequences (GCS) and second order Reed-Muller (RM) codes that the large sets of binary length Golay complementary pairs can be obtained from certain second-order cosets of the classical first order RM code. More precisely the 2^h-PSK Golay sequences can be obtained from certain cosets of the first order Reed-Muller codes within the second order Reed-Muller codes over the integer ring $\mathbb{Z}_{2^h}$.

The rest of the paper is organized as follows. Section II and III describes some background material on OFDM and PAPR. In section IV Golay complementary sequences and their properties are reviewed. In section V construction of 16-QAM OFDM sequences is discussed and its PAPR is calculated. Finally the paper is then concluded in VI.

II. OFDM SYSTEM MODEL

A simple model of OFDM system is shown in Figure 1 as:

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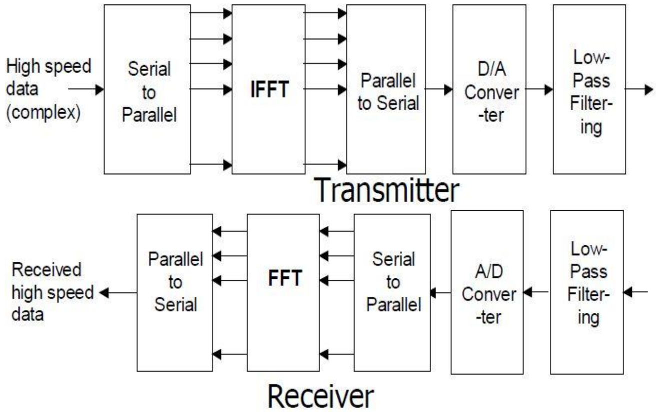

Fig.1. Baseband OFDM System

In OFDM a high rate data stream is divided into many low data streams i.e. a serial stream of data is converted to N parallel streams. These binary data streams are then mapped to form digital symbols using modulation techniques such as BPSK, QPSK, and QAM etc. These mapped symbols are then superimposed onto the orthogonal carriers in IFFT block. A composite signal so formed by multiplexing these modulated signals is called the OFDM signal. Such a scheme has several advantages like easier frequency-domain equalization, low sensitivity to impulse noise, good modularity etc. Moreover, it is possible to choose the constellation size and energy for each subcarrier, thus approaching the theoretical capacity of the channel.

Mathematically the transmitted OFDM signal can be represented as the real part of the complex signal and is given as [8]:

$$S(t)=\sum_{i=0}^{n-1} c_i(t) e^{2j\pi f_i t} \quad (1)$$

Where f_i is the frequency of the ith carrier, $c_i(t)$ is constant over a symbol period of duration T. To maintain orthogonality, the carrier frequencies are related by $f_i = f_0 + i\Delta f$ where f_i represents the frequency of the ith carrier, f_0 is the smallest carrier frequency and Δf denotes the integer multiple of the OFDM symbol rate.

Let $c_i(t)$ takes the value c_i over a given symbol period, then the corresponding OFDM signal is denoted by $S_c(t)$ and can be expressed as

$$S_c(t)=\sum_{i=0}^{n-1} c_i e^{2j\pi f_i t} \quad (2)$$

Instantaneous envelope power associated with the sequence $S_c(t)$ is given by $P_c(t)$

$$P_c(t)=\left|S_c(t)\right|^2 = S_c(t).S_c^*(t) \quad (3)$$

III. PAPR PROBLEM

Although OFDM is very effective and efficient technique in the field of high speed multi-carrier data transmission, it suffers from a drawback of high PAPR (Peak to Average Power Ratio) that occurs due to the time-domain superposition of many data subcarriers, and thus the resulting time domain signal exhibits the Rayleigh-like characteristics and large time domain amplitude variations [4]. When this high PAPR signals are amplified by practical power amplifiers that have non-linear response and finite amplitude ranges, it results in the non-linear distortions on the communication. These non-linear distortions create out-of-band distortion which interferes with co-channel users, and in-band-distortion that causes self-interference. To overcome these non- linear distortions, transmit power amplifier has to be operated in its linear region with a large dynamic range but linear Power Amplifiers are power inefficient and expensive too. In fact high cost of practical networks is due to use of high power linear amplifiers. Also large PAPR demands for D/A converters with large dynamic range to accommodate the large peaks of the OFDM signals and high precision DAC supports high PAPR but at higher cost. Thus large PAPR increases the complexity of D/A and A/D converters. Therefore reduction of Peak to Average Power Ratio is of major concern [3].

Instantaneous envelope power associated with signal is given as

$$P_c(t)=\left|S_c(t)\right|^2 = S_c(t).S_c^*(t) \quad (4)$$

Also the average power of c can be expressed as

$$\frac{1}{T}\int_0^T P_c(t)\,dt = \|c\|^2 = \sum_{k=0}^{n-1}|c_k|^2$$

Where, $\|c\|^2 = \sum_{k=0}^{n-1}|c_k|^2$

Mathematically PAPR is defined as the ratio between the instantaneous power of the peaks and the average power of the signal. Thus

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$$\text{PAPR}(c) = \frac{\max_{0\le t\le T} P_c(t)}{P_{av}} \quad (4)$$

$$\text{Crest Factor}=\sqrt{\text{PAPR}}$$

We would like to design codes C such that $PAPR\ (C) = \max_{C\in c} PAPR\ (c)$ are small. PAPR is usually analyzed by using statistical parameter called complementary cumulative distribution function (CCDF). CCDF shows the probability that PAPR is above a threshold level. CCDF is defined as

$$\text{P(PAPR} \ge \text{Z)=1-((1-exp(z))}^N$$

Where, N is the number of subcarriers.

IV. GOLAY COMPLEMENTARY SEQUENCES

More than sixty years ago, efforts by Marcel Golay led to the discovery of complementary sequences which were later named after him as Golay complementary sequences [4]. The perfect autocorrelation property of GCS has proved to be of value in a variety of applications. Golay complementary pairs have the property that the sum of their autocorrelation functions vanishes at all delays other than zero. Two sequences are said to be Golay complimentary pair if sum of their autocorrelation functions vanishes at all delays other than zero [5]. The aperiodic autocorrelation of the sequence 'c' is defined by

$$A_c(u) = \sum_{i=0}^{n-1} c_i c_{i+u}^*$$

Consider two sequences 'a' and 'b' of length 'n', $a=(a_0,a_1,...,a_{n-1}), b=(b_0,b_1,...,b_{n-1})$

Where, $a_i, b_i \in \mathbb{Z}_4$.

Where, $\mathbb{Z}_h = (0,1,...,h-1)$ is an integer ring of size h. These two sequences are said to be Golay complimentary pair if sum of their aperiodic autocorrelation is a delta function [6]. Thus

$$A_a(u) + A_b(u) = \left(\|a\|^2 + \|b\|^2\right)\delta(u) \quad (5)$$

$$\delta(u) = \begin{cases} 1\ , \text{ if } u = 0 \\ 0\ , \text{ if } u \ne 0 \end{cases}$$

$\delta(.)$ = Kronecker function

Any sequence which is the member of the Golay complementary (GCP) pair is called a Golay complementary sequence (GCS). Therefore,

$$A_a(u) + A_b(u) = \begin{cases} 0, \text{ at } u \ne 0 \\ 2n, \text{ at } u = 0 \end{cases}$$

$c = [c_k]_{k=0}^{n-1}$ forms the Fourier series pair, their conjugates also form a Fourier pair. Also, $A_c(u)$ and $P_c(t)$ forms the Fourier transform pair, therefore aperiodic autocorrelation can be studied in frequency domain to obtain the knowledge about $P_c(t)$ in time domain.

Theorem 1. Let a be one of the sequence which belongs to a complementary pair. Then $PAPR(a) \le 2$.

Proof: Since $P_a(t) + P_b(t)$ and $A_a(u) + A_b(u) = \left(\|a\|^2 + \|b\|^2\right)\delta(u)$ form a pair of Fourier series, taking Fourier transform we have $P_a(t) + P_b(t) = \|a\|^2 + \|b\|^2$, $0 \le t \le T$. For PSK modulation of unit energy, the average power $\|c\|^2$ of any sequence c is equal to $\sum_{k=0}^{n-1} |c_k|^2 = n$. Since $P_a(t)$ and $P_b(t)$ are non-negative and, $P_a(t) + P_b(t) = 2n$ thus we have $PAPR(c) \le \frac{2n}{n} = 2$. So it is concluded that PAPR is upper bound by 2 for any Golay sequence c in PSK modulation [7].

V. CONSTRUCTION OF 16-QAM OFDM SEQUENCES

Rößing and Tarokh [9], Chong et.al [10] demonstrated a construction of 16-QAM sequences from QPSK Golay complementary sequences, and derived bounds for the PMEPR of the 16-QAM sequences. They observed that any 16-QAM symbol can be decomposed uniquely into a pair of QPSK symbols because any point on the 16-QAM constellation can be written as

$$S_{16-QAM} = \frac{1}{\sqrt{2}} S_{QPSK} + \sqrt{2} S_{QPSK} \quad (9)$$

So, the construction 16-QAM Golay sequences requires two QPSK symbols where each symbol can have four phase

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values as coefficients, that is fourth roots of unity; so as to get a total of 16 constellation points. Let $S_{16-\mathrm{QAM}}$ denotes the 16-QAM constellation symbols and can be written as the sum of two QPSK symbols [9]. Figure below shows the constellation diagram of 16-QAM as the sum of two QPSK symbols.

Consider S_{QPSK} be the set of QPSK constellation symbols and can be represented as $S_{QPSK} = \left\{ e^{\frac{j\pi}{4}}, je^{\frac{j\pi}{4}}, -e^{\frac{j\pi}{4}}, -je^{\frac{j\pi}{4}} \right\}$

Let $c = (c_0, c_1, ..., c_{n-1})$, $c_i \in S_{16-QAM}$ be the 16-QAM sequence and can be associated with two QPSK sequences as

$$x = (x_0, x_1, ..., x_{n-1}) \in \mathbb{Z}_4^n,$$

$$y = (y_0, y_1, ..., y_{n-1}) \in \mathbb{Z}_4^n$$

Figure 2 below shows the constellation diagram of 16-QAM as the sum of two QPSK symbols

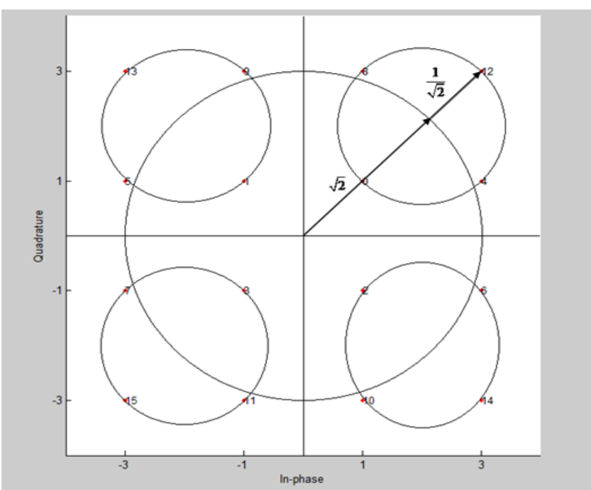

Fig. 2. Constellation diagram of 16-QAM as sum of two QPSK symbols

$$S_{16-QAM} = \tfrac{1}{\sqrt{2}} e^{\frac{j\pi}{4}} j^x + \sqrt{2} e^{\frac{j\pi}{4}} j^y \qquad (9)$$

$$= e^{\frac{j\pi}{4}} \left(\tfrac{1}{\sqrt{2}} j^x + \sqrt{2} j^y \right)$$

$$c_k = e^{\frac{j\pi}{4}} \left(\tfrac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} \right) \qquad 0 \le k \le n-1$$

Let $S_x(t)$ and $S_y(t)$ be the two QPSK OFDM symbols, then

$$S_x(t) = \sum_{k=0}^{n-1} e^{\frac{j\pi}{4}} j^{x_k} . e^{\frac{j2\pi kt}{T}} \qquad (10)$$

$$S_y(t) = \sum_{k=0}^{n-1} e^{\frac{j\pi}{4}} j^{y_k} . e^{\frac{j2\pi kt}{T}} \qquad (11)$$

A 16-QAM OFDM symbol $S_c(t)$ can be written as the sum of the two QPSK OFDM signals.

$$S_c(t) = \sum_{k=0}^{n-1} c_k . e^{\frac{j2\pi kt}{T}}$$

$$S_c(t) = \sum_{k=0}^{n-1} \left(\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} \right) e^{\frac{j\pi}{4}} . e^{\frac{j2\pi kt}{T}}$$

$$S_c(t) \quad \frac{1}{\sqrt{2}} \sum_{k=0}^{n-1} j^{x_k} e^{\frac{j\pi}{4}} . e^{\frac{j2\pi kt}{T}} + \sqrt{2} \sum_{k=0}^{n-1} j^{y_k} e^{\frac{j\pi}{4}} . e^{\frac{j2\pi kt}{T}}$$

From (10) and (11), we can write

$$S_c(t) = \frac{1}{\sqrt{2}} S_x(t) + \sqrt{2} S_y(t)$$

Let x and y be the two Golay complementary sequences of length 'n'. Thus instantaneous envelope power of a 16-QAM OFDM signal $S_c(t)$ is bounded above by

$$P_c(t) = |S_c(t)|^2$$

$$P_c(t) = \left| \frac{1}{\sqrt{2}} S_x(t) + \sqrt{2} S_y(t) \right|^2$$

$$\le \left| \frac{1}{\sqrt{2}} . \sqrt{2n} \right|^2 + \left| \sqrt{2} . \sqrt{2n} \right|^2 + 2 . \frac{1}{\sqrt{2}} . \sqrt{2n} . \sqrt{2} . \sqrt{2n}$$

$$\therefore P_c(t) = 9n \qquad (12)$$

The minimum Euclidean distance between signal points of 16-QAM constellation is assumed to be *E* = 1. Thus for each

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signal point with equal probability, the average energy of 16-QAM constellation $c_k = \sum_{k=0}^{n-1} |c_k|^2$ can be calculated as

$$= \left[\sqrt{\left(\frac{1}{2}\right)^2 + \left(\sqrt{2}\right)^2} \right]^2 = 2.5 \qquad (13)$$

Average power is $P_{av} = 2.5n$ (14)

By taking the values from (12), (14) and putting in (4), we get

$$\text{PAPR} = \frac{\text{max.power}}{\text{avg.power}} = \frac{9n}{2.5n} = 3.6$$

$$\text{PAPR(dB)} = 5.56\text{(dB)}$$

Therefore PAPR for 16-QAM OFDM signal is bounded above by 3.6. So, for a 16-QAM OFDM code we can trade off the size, the PAPR, and the squared Euclidean distance of an OFDM code by an appropriate selection of two QPSK component codes.

I. CONCLUSION

This paper presents a method for the construction of M-QAM sequences particularly 16-QAM from QPSK sequences. By using Golay sequences the Peak to Average Power Ratio is bounded by 5.56dB. Using the construction discussed in the paper it is possible to increase the transmission rate as compared to 4-PSK transmission. Also, the approach augmented in the archive can be applied to generate low peak power OFDM sequences defined over 8-QAM, 32-QAM, 64-QAM and 256-QAM constellations, as these constellations can also be written as the set sums of 4-PSK and BPSK constellations.

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