

Kalman Filter

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Abstract: The Kalman filter is a Recursive algorithm and recursive means doesn't need to store all previous measurements and reprocess all data each time step. The Kalman filter is the best possible (optimal) estimator for a large class of problems and a very effective and useful estimator for an even larger class. With a few conceptual tools, the Kalman filter is actually very easy to use.

1. INTRODUCTION

Stochastic Estimation

There are many application-specific approaches to "computing" (estimating) an unknown state from a set of process measurements. For example, consider our work in tracking for interactive computer graphics. While the requirements for the tracking information vary with application, the fundamental source of information is the same: pose estimates are derived from noisy electrical measurements of mechanical, inertial, optical, acoustic, or magnetic sensors. This noise is typically statistical in nature (or can be effectively modeled as such), which leads us to stochastic methods for addressing the problems.

Within the significant toolbox of mathematical tools that can be used for stochastic estimation from noisy sensor measurements, one of the most well-known and often-used tools is what is known as the Kalman filter. The Kalman filter is named after Rudolph E. Kalman, who in 1960 published his famous paper describing a recursive solution to the discrete-data linear filtering problem. The Kalman filter is essentially a set of mathematical equations that implement a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance—when some presumed conditions are met. The Kalman filter has been used extensively for tracking in interactive computer graphics.

2. KALMAN FILTER (CONCEPTUAL VIEW)

Make prediction based on previous data

↓
Take measurement

Optimal estimate (\hat{y}) = Prediction + (Kalman Gain) * (Measurement - Prediction)

Variance of estimate = Variance of prediction * (1 - Kalman Gain)

* Initial conditions (\hat{y}_{k-1} and σ_{k-1})

* Prediction (\hat{y}_k, σ_k)

• Make initial conditions and model to make prediction.

* Measurement (z_k)

* Correction (\hat{y}_k, σ_k)

• Use measurement to correct prediction by 'blending' prediction and residual – always a case of merging only two Gaussians

• Optimal estimate with smaller variance

Blending Factor

- If we are sure about measurements:
- Measurement error covariance (R) decreases to zero
- K increases and weights residual more heavily than prediction
- If we are sure about prediction
- Prediction error covariance P_k decreases to zero
- K decreases and weights prediction more heavily than residual

Assumptions behind Kalman Filter

- The model you use to predict the 'state' needs to be a LINEAR function of the measurement
- The model error and the measurement error (noise) must be Gaussian with zero mean.

3. THE DISCRETE KALMAN FILTER

Kalman Filter where the measurements occur and the state is estimated at discrete points in time.

1. The Process to be Estimated

The Kalman filter addresses the general problem of trying to estimate the state $x \in \mathcal{R}^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_k = Ax_{k-1} + Bu_k + w_{k-1}, \quad (1)$$

with a measurement $z \in \mathcal{R}^m$ that is

$$z_k = Hx_k + v_k. \quad (2)$$

The random variables w_k and v_k represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions

$$p(w_k) \sim N(0, Q), \quad (3)$$

$$p(v_k) \sim N(0, R). \quad (4)$$

In practice, the process noise covariance Q and measurement noise covariance R matrices might change with each time step or measurement.

2. The Computational Origins of the Filter

We define $\hat{x}_k^- \in \mathcal{R}^n$ (note the "super minus") to be our a priori state estimate at step k given knowledge of the process prior to step k, and $\hat{x}_k \in \mathcal{R}^n$ to be our a posteriori

state estimate at step k given measurement z_k . We can then define a priori and a posteriori estimate errors as

$$e_k^- \equiv x_k - \hat{x}_k^-$$

and $e_k^+ \equiv x_k - \hat{x}_k^+$

The a priori estimate error covariance is then

$$P_k^- = E[e_k^- e_k^{-T}], \quad (5)$$

and the a posteriori estimate error covariance is

$$P_k^+ = E[e_k^+ e_k^{+T}]. \quad (6)$$

In deriving the equations for the Kalman filter, we begin with the goal of finding an equation that computes an a posteriori state estimate \hat{x}_k^+ as a linear combination of an a priori estimate \hat{x}_k^- and a weighted difference between an actual measurement z_k and a measurement prediction $H\hat{x}_k^-$ as shown below in equation (7).

“The Probabilistic Origins of the Filter”

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \quad (7)$$

3. THE PROBABILISTIC ORIGINS OF THE FILTER

The justification for equation (7) is rooted in the probability of the a priori estimate \hat{x}_k^- conditioned on all prior measurements z_k (Bayes' rule). For now let it suffice to point out that the Kalman filter maintains the first two moments of the state distribution,

$$E[x_k] = \hat{x}_k$$

$$E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] = P_k$$

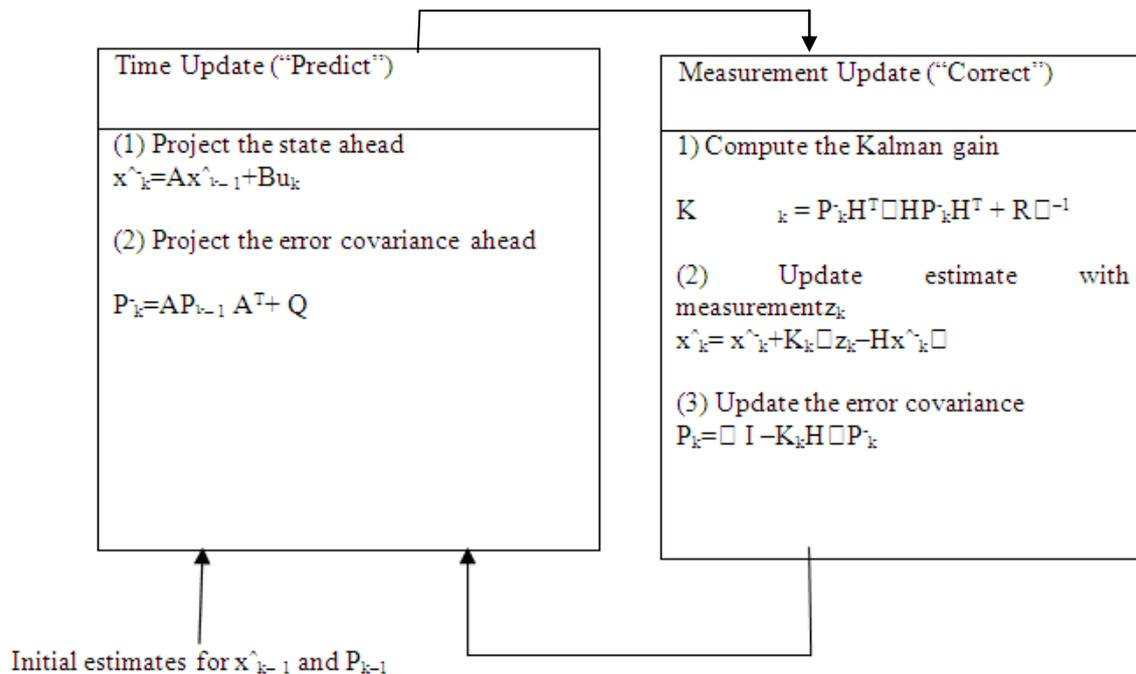
4. THE DISCRETE KALMAN FILTER ALGORITHM

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback—i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations.

After each time and measurement update pair, the process is repeated with the previous a posteriori estimates used to project or predict the new a priori estimates. This recursive nature is one of the very appealing features of the Kalman filter—it makes practical implementations much more feasible than an implementation of a Wiener filter which is designed to operate on all of the data directly for each estimate. The Kalman filter instead recursively conditions the current estimate on all of the past measurements.

A complete picture of the operation of the Kalman filter



4. CONCLUSION

Kalman Filter is the best algorithm for tracking objects with correct approximation. In this paper, we described Kalman Filter and advanced version of Kalman Filter which we can use for tracking objects.

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BIOGRAPHY

Nivedita is now pursuing M. Tech in computer science and engineering. She is the author of many articles. Her research interests are Mobile Ad hoc Networks and Sensor Networks. And she is always supported by her father.