

Denoising of an Image using the New Contourlet Transform

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Abstract: The Contourlet Transform has an efficient filter bank construction and low redundancy which makes it an impressive computational tool for different applications in image processing. It provides a directional multiresolution image representation, which is capable of capturing and representing singularities along smooth object boundaries in natural images. But a major disadvantage of the original Contourlet Transform is that, its basis images are not localized in the frequency domain. Here, a new Contourlet Transform is proposed as a solution. In this multiscale pyramid (which is defined in the frequency domain) is used against Laplacian pyramid, which is used in Counterlet Transform. It is observed that the resulting basis images are sharply localized in the frequency domain and exhibit smoothness along their main ridges in the spatial domain. Using Image Denoising, it can be shown that the proposed New Contourlet Transform can significantly outperform the original Transform both in terms of PSNR (by several dB's) and in visual quality.

Keywords: Contourlet Transform, Multiscale Pyramid, Directional Filter Banks, Image Denoising.

I. INTRODUCTION

One of the most important tasks in image processing is studying and exploiting the special properties of natural images. The main distinguishing feature of natural images is that they have intrinsic geometrical structures such as along object boundaries.

Previous work in Contourlet Transform, Do and Vetterli [1] proposed the Contourlet Transform as a directional multiresolution image representation that can efficiently capture and represent smooth object boundaries in natural images. The Contourlet Transform is constructed as a combination of the Laplacian pyramid [2] and the Directional Filter Banks (DFB) [3]. Conceptually, the flow of operation can be illustrated by Figure 1, where the Laplacian pyramid iteratively decomposes a 2-D image into lowpass and highpass subbands, and the DFB are applied to the highpass sub-bands to further decompose the frequency spectrum.

Basically the non-ideal filters are used, but the resulting Contourlets do not have the desired shape frequency domain localization. Although the majority of the energy in each subband is still concentrated on the ideal support regions, there are also significant amount of aliasing components showing up at locations far away from the desired support. This kind of frequency aliasing is undesirable, since the resulting Contourlets in the spatial domain are not smooth along their main ridges and exhibit some fuzzy artifacts.

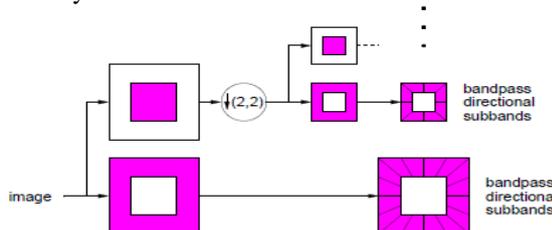


Fig.1: The original Contourlet Transform with the resulting frequency division

A pictorial explanation of the cause of this frequency non-localization problem is explained as a solution in section II. The new construction of the Contourlet Transform, in which the non-localization problem is greatly alleviated, is discussed in Section III. The resulting Denoised images are presented in Section IV to confirm the superiority of the proposed new construction over the original transform. Conclude the paper in Section V.

II. CONTOURLETNON-LOCALIZATION PROBLEM

The Contourlet Transform with two levels of multiscale decomposition, followed by angular decomposition is shown in Figure 2. This is the block diagram of the Contourlet Transformation. In this, the Laplacian pyramid shown in the diagram is a simplified version of its actual implementation. Nevertheless, this simplification serves our explanation purposes satisfactorily. By using the multirate identities, rewrite the filter bank into its equivalent parallel form, as shown in the right part of Figure 2. In the following discussions, concentration has been made on channel 2 of the filter bank.

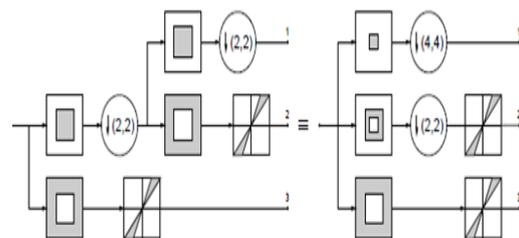


Fig. 2: Block diagram of the Contourlet Transform with two levels of multiscale decomposition. Gray regions represent the ideal passband support of the component filters. Left: The iterated form. Right: The equivalent parallel form.

A more realistic illustration of one of the directional filters from the DFB has been shown, when non-ideal filters are used. Gray regions in the figure represent the ideal passband, and patterned regions represent the aliasing areas concentrated along two parallel lines ($\omega_2 = \pm\pi$).

Two reasons contribute to this aliasing effect. The first one is due to the periodicity of 2-D frequency spectrums of discrete signals. In Figure 3(a), the patterned regions marked by p are actually the transition bands of the wedge-shaped filters, folded back through 2π periodization. The other reason is intrinsic to the frequency partitioning of the DFB. Using the argument of permissible passband supports proposed by Chen and Vaidyanathan [5], in that perfect reconstruction and frequency domain localization cannot be achieved simultaneously by a critically-sampled filter bank with the frequency partitioning of the DFB. In other words, since the DFB are critically-sampled and have perfect reconstruction, their component filters must exhibit aliasing components outside the desired passband regions.

When the DFB is combined with a multiscale decomposition as in the Contourlet Transform, the aliasing problem becomes a serious issue. For instance, to calculate the equivalent filter of the second channel in Figure 2, the directional filter must first be upsampled by 2 along each dimension, as shown in Figure 3(b). As a result of the upsampling, the aliasing components (again represented by the patterned-regions in the figure) are folded towards the lowpass regions and are concentrated mostly along two lines ($\omega_2 = \pm\frac{\pi}{2}$). Combining the upsampled DFB with the bandpass filter as shown in Figure 3(c), results in the Contourlet subband filter for channel 2 as shown in Figure 3(d), and then the Contourlets are not localized in frequency, with substantial amount of aliasing components outside the desired trapezoid-shaped support.

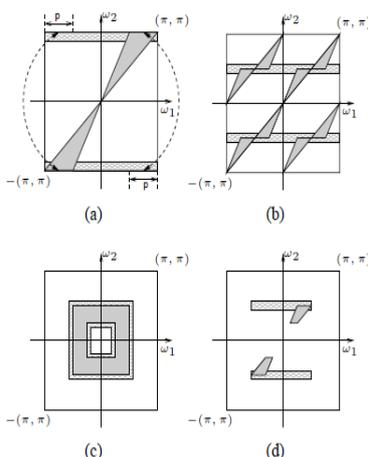


Fig. 3: Illustration of the frequency domain aliasing problem of the Contourlet Transforms. Gray regions represent the ideal passband support. Patterned regions represent the aliasing components or transition bands. (a) One directional filter. (b) The directional filter after being upsampled by 2 along each dimension. (c) A bandpass filter from the Laplacian pyramid. (d) The resulting Contourlet subband.

III. A NEW PROPOSED CONTOURLET WITH FREQUENCY LOCALIZATION

Here, the new construction of the Contourlet Transform is proposed. DFB for directional decomposition is still used. However, an important distinction from the original Contourlet Transform is that, instead of using the Laplacian pyramid, the new pyramid structure for the multiscale decomposition is used. In the diagram, we use $L_i(\omega)(i = 0, 1)$ to represent the lowpass filters and $D_i(\omega)(i = 0, 1)$ to represent the highpass filters in the multiscale decomposition, with ω def = (ω_1, ω_2).

The DFB is attached to the highpass branch at the finest scale and bandpass branches at all coarser scales.

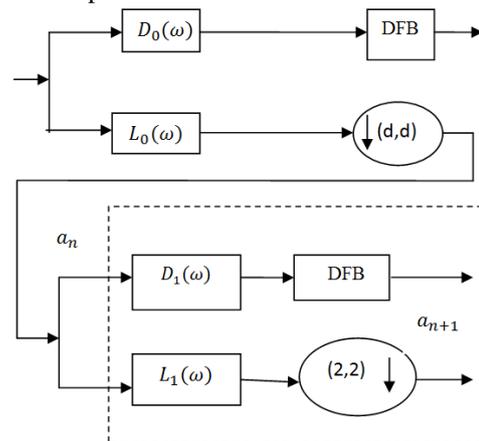


Fig. 4: The block diagram of the New Contourlet Transform. Only the analysis part is shown, while the synthesis part is exactly symmetric.

The lowpass filter $L_0(\omega)$ in the first level is downsampled by d along each dimension, with d being a number to be determined shortly, and the lowpass filter $L_1(\omega)$ in the second level is downsampled by $(2, 2)$. To have more level of decomposition, we can recursively insert at point a_{n+1} a copy of the diagram contents enclosed by the dashed rectangle.

An important difference from the Laplacian pyramid as shown in Figure 2, the new multiscale pyramid can employ a different set of lowpass and highpass filters for the first level and all other levels. As will be seen shortly, this is a crucial step in reducing the frequency-domain aliasing of the DFB. Here we specify the lowpass filters $L_i(\omega)$ ($i = 0, 1$) in the frequency domain as $L_i(\omega) = L_i^{1d}(\omega_1) \cdot L_i^{1d}(\omega_2)$, and $L_i^{1d}(\omega)$ is a 1-D lowpass filter with passband frequency $\omega_{p,i}$ and stopband frequency $\omega_{s,i}$ and a smooth transition band, defined as

$$L_i^{1d}(\omega) = \begin{cases} 1 & \text{for } |\omega| \leq \omega_{p,i} \\ \frac{1}{2} + \frac{1}{2} \cos\left(\frac{(|\omega| - \omega_{p,i})\pi}{\omega_{s,i} - \omega_{p,i}}\right) & \text{for } \omega_{p,i} < |\omega| < \omega_{s,i} \\ 0 & \text{for } \omega_{s,i} \leq |\omega| \leq \pi \end{cases} \dots\dots(1)$$

For $|\omega| \leq \pi$ and $i = 0, 1$.

Assuming that, the aliasing introduced by the downsampling operations can be completely cancelled, we can simplify the perfect reconstruction condition for the multiscale pyramid as

$$|L_i(\omega)|^2 + |D_i(\omega)|^2 \equiv 1, \quad \text{for } i = 0, 1. \dots\dots(2)$$

Once specified, the lowpass filters, the highpass filters $D_i(\omega)$ can be obtained from (2) to ensure perfect reconstruction.

IV. RESULTS OF THE NEW CONSTRUCTION OVER THE OLD CONTOURLET

In this section, “Contourlet” is used to denote the original transform and the “New Contourlet” to denote the different variants of the proposed New Contourlet Transform, with the numbers corresponding to their respective redundancy ratios.

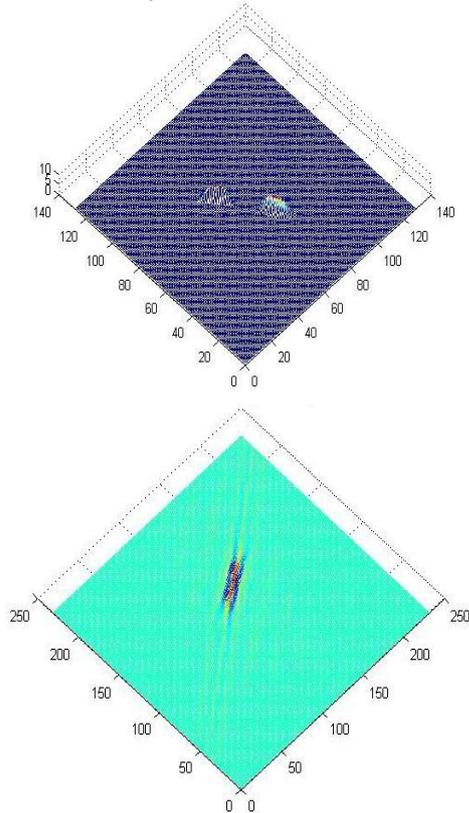


Fig. 5: Contourlet of Basis images. (a) Frequency Domain. (b) Spatial Domain

A. Basis Images

In Figure 5, the frequency and spatial domain basis images of the New Contourlet Transform proposed in this paper are shown. Figure 5(a) shows that, the original Contourlet Transform suffers from the frequency non-localization problem. In sharp contrast, the new construction produce basis images that are well-localized in the frequency domain, as shown and in Figure 5(b), the improvement in the frequency localization is also reflected in the spatial domain. So the spatial regularity of Contourlets can be greatly improved by using the new construction.

Table 1: PSNR values of the Denoised images

$\bar{\sigma}$ (Redundancy)	IMAGE		
	30	40	50
Contourlet	26.8768	25.6851	24.6779
New Contourlet	28.822	27.6917	26.7154

B. Denoising

In this experiment, the Denoising performance of the proposed New Contourlet Transform with that of the original transform has been compared by using the standard hard thresholding denoising method..

Table 1 shows the PSNR (in dB) of the denoised images by using different transforms. Although New Contourlet has the same redundancy ratio and similar computational cost as the original Contourlet Transform, it outperforms the latter by more than 2 dB.

Comparison of Denoised “Lena” images by using Contourlet and New Contourlet is shown in Fig.6



New Contourlet PSNR=28.882, at $\bar{\sigma}$ =30



New Contourlet PSNR=27.6917, at $\bar{\sigma}$ =40.

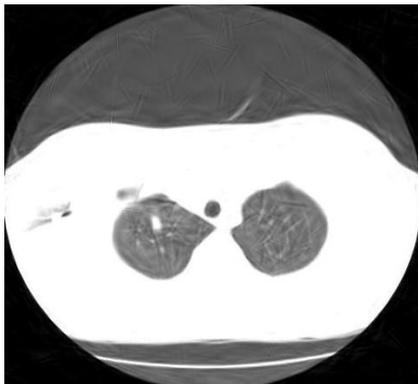


Contourlet PSNR=26.8768, at $\bar{\sigma}$ =30

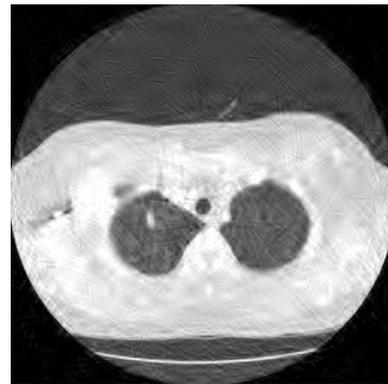


Contourlet PSNR=25.6851, at $\bar{\sigma}$ =40.

Fig. 6: Comparison of Denoised “Lena” images by using Contourlet and New Contourlet.



New Contourlet PSNR=30.4665, $\sigma=30$



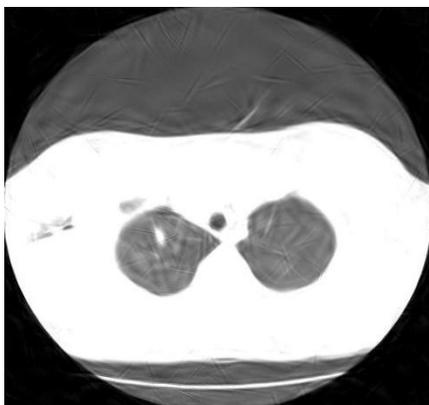
Contourlet PSNR=26.9247, $\sigma=40$.



New Contourlet PSNR=29.3485, $\sigma=40$



Contourlet PSNR=26.2658, $\sigma=50$



New Contourlet PSNR=28.4255, $\sigma=50$



Contourlet PSNR=28.6661, $\sigma=30$

Fig.7 shows Comparison of Denoised CT Lung images by using Contourlet and New Contourlet.

V. CONCLUSION

In this paper, a new construction for the Contourlet Transform has been proposed. Compared with the old version, the new construction produces basis images with much better localization in the frequency domain and regularity in the spatial domain. In applications such as image denoising, we have shown that the proposed New Contourlet construction significantly outperforms the original transform.

ACKNOWLEDGMENT

The authors would like to acknowledge and thank the professors from their college, especially the guide for the project, **Mr. Niladri Shekhar Mishra**, for his support and contribution. Also, the authors wish to express their gratitude to their respective parents for their support through the course of the project, and their prayers helped to complete the project on time.

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