Recursive filter Based Channel Estimation and Correction

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ABSTRACT: Communications through frequency-selective fading channels suffer inter-symbol interference (ISI), which limits the data rate. Complex equalizers are usually needed to compensate for the channel distortion. Orthogonal Frequency Division Multiplexing (OFDM) divides the channel spectrum into many sub-bands, each of which carries low-rate data. Since each sub-channel is narrow band, communications through it experience only flat-fading. The sub-channels are separated with the minimum space required by channel Orthogonality. Therefore, a large number of low-rate data streams can be transmitted in parallel and aggregate to a high-rate one. Channel estimation and prediction algorithms are developed and evaluated for use in broadband adaptive OFDM downlinks over fading channels for vehicular users. Accurate channel estimation may be obtained by using a combined pilot aided and decision-directed approach based on Recursive filtering and prediction. The correlation properties of the channel in both time and space are taken into account. Kalman performance at much lower computational complexity is attained with recently developed constant gain adaptation laws. We present and evaluate a state-space realization of such an adaptation law, with computational complexity of the order of the square of the number of parallel tracked pilot subcarriers. In an adaptive OFDM system, prediction of the channel power a few milliseconds ahead will also be required. Frequency-domain channel estimates can be transformed to the time domain, and used as regressors in channel predictors based on linear regression. We also make a preliminary evaluation of the direct use of complex channel prediction in the frequency domain for channel power prediction.

Keywords: OFDM (Orthogonal Frequency Division Multiplexing), Orthogonality, Channel Estimation Recursive Filter, Fading Environment, Adaptive Modulation

I. INTRODUCTION

Adaptive transmission can radically improve the spectral efficiency when multiple users have independently fading links. The users may then share the available bandwidth, and resources are allocated to terminals who best and/or can need utilize them best via link adaptation. This paper focuses on the downlink of an adaptive OFDM system that employs Frequency Division Duplex (FDD). The aim is to attain high spectral efficiency for wide area coverage and to serve also vehicular users, with velocities around 100 km/h. In this adaptive OFDM downlink, packet data streams to a number of active users are multiplexed on a common bandwidth. Each user must estimate and predict the channel over the whole utilized bandwidth, and report which parts of the spectrum will have the best signal to interference ratio. A scheduler, located at the base station, then allocates time frequency resources based on the requirements and channel qualities of each user [1].

A. System Model

An OFDM system is a multiplexing of several (N) digital modulation systems over a bandwidth of W Hz in a baseband form. The modulation filters of the sub-systems are rectangular pulses modulated on the carriers of frequencies

\[
\phi_k(t) = \begin{cases} 
\frac{1}{\sqrt{T - T_{cp}}} & \text{if } t \in [0,T] \\
0 & \text{otherwise}
\end{cases}
\]

(2)

where \( T = NT_s + T_{cp} \) is the symbol duration, \( T_s = 1/W \), and \( T_{cp} \) is the length of the guarding signals called cyclic prefix (CP).
Channel estimation in OFDM systems has been well studied. Based upon whether those channel estimation algorithms apply training symbols, we can divide those algorithms into three categories: training (pilot) based algorithms, blind algorithms and semi-blind algorithms. We will discuss these three groups of algorithms separately. Training-based algorithms assume known symbols (training or pilot symbols) are inserted in the transmitted signals. It is then possible to identify the channel at the receiver through exploiting knowledge of these known symbols. Blind algorithms estimate the channel based on properties of the transmitted signals. Semi-blind algorithms can improve the performance of blind algorithms by exploiting the knowledge of both known symbols and properties of the transmitted signals. The objective of semi-blind channel estimation algorithms is to get better performance than blind algorithms while requiring fewer known symbols than training based channel estimation algorithms. The channel estimation algorithms proposed in are training based algorithms. The idea of the algorithms is to exploit OFDM channel correlation, thus the statistics (mean, variance) of the channel must be known to make these algorithms work. In channel statistics are not needed. The idea is to build a model with unknown coefficients, then exploit the information of the output symbols and training symbols to estimate the coefficients of the model.

The MMSE algorithm uses only the correlation of the channel in frequency domain (i.e., the correlation between sub-channels) and fails to address time-domain dynamics. The MMSE algorithm performs better than LS algorithm at the cost of higher complexity. Additionally, MMSE algorithm requires knowledge of channel covariance and noise variance, which are assumed to be known as a priori knowledge. The LS algorithm does not need any knowledge about the channel. In MMSE OFDM channel estimation algorithm in which coarse channel estimates from several successive OFDM symbols are further combined optimally in the MMSE sense to get an updated channel estimate. The algorithm exploits both time domain and frequency domain channel correlation, and makes use of the fact that the OFDM channel correlation can be written as the product of time domain channel correlation and frequency domain channel correlation. The proposed MMSE channel estimator first exploits the frequency domain channel correlation, and then exploits the time domain channel correlation. Information of channel statistics and operating SNR are needed to make this algorithm work. Moreover, the proposed MMSE algorithm can work in a mismatch mode. However, its performance degrades much if the Doppler frequencies and delay spreads applied in the MMSE algorithm are smaller than the ones of the practical channel. However, the initial coarse channel estimates are obtained independently from one OFDM symbol to another without taking advantage of the time domain dynamics of the channel.

The time varying channel is approximated as a Gauss-Markov process, and a Kalman filter is used for updating channel states in time domain. For a fixed percentage of pilots, assuming pilot symbols are periodically sent through L+1. Sub-channels, the goal is to optimize the
placement of pilot symbols by minimizing the average steady state MSE of the channel estimator. It is shown that single pilot periodic placement achieves the minimum MSE with the assumption that channel fading is relatively slow compared to the symbol rate, i.e. the channel state remains unchanged over the duration of each OFDM block, but changes from block to block.

III. PROPOSED SYSTEM

All paragraphs must be indented. All paragraphs must be justified, i.e. both left-justified and right-justified.

A. OFDM Systems

After the subcarrier modulation stage each of the data subcarriers is set to amplitude and phase based on the data being sent and the modulation scheme all unused subcarriers are set to zero. This sets up the OFDM signal in the frequency domain [2]. An IFFT is then used to convert this signal to the time domain, allowing it to be transmitted. In the frequency domain, before applying the IFFT, each of the discrete samples of the IFFT corresponds to an individual subcarrier. Most of the subcarriers are modulated with data. The outer subcarriers are unmodulated and set to zero amplitude. These zero subcarriers provide a frequency guard band before the nyquist frequency and effectively act as an interpolation of the signal and allows for a realistic roll off in the analog anti-aliasing reconstruction filters.

B. RF Modulation

The output of the OFDM modulator generates a base band signal, which must be mixed up to the required transmission frequency. This can be implemented using analog techniques as shown in Figure 2 or using a Digital up Converter as shown in Figure 3. Both techniques perform the same operation, however the performance of the digital modulation will tend to be more accurate due to improved matching between the processing of the I and Q channels, and the phase accuracy of the digital IQ modulator.

C. Adaptive Modulation

Adaptive modulation is a powerful technique for maximizing the data throughput of subcarriers allocated to a user. Adaptive modulation involves measuring the SNR of each subcarrier in the transmission, then selecting a modulation scheme that will maximize the spectral efficiency, while maintaining an acceptable BER. This technique has been used in Asymmetric Digital Subscriber Line (ADSL) to maximize the system throughput. ADSL uses OFDM transmission over copper telephone cables. The channel frequency response of copper cables is relatively constant and so reallocation of the modulation scheme does not need to be performed very often, as a result the benefit greatly out ways the overhead required for measuring of the channel response [3]. Using adaptive modulation in a wireless environment is much more difficult as the channel response and SNR can change very rapidly, requiring frequent updates to track these changes. Adaptive modulation has not been used extensively in wireless applications due to the difficulty in tracking the radio channel effectively. The effectiveness of a multiuser OFDM system using an adaptive subcarrier, bit and power allocation was investigated. Optimization of the transmission was achieved by minimizing the power requirement for a given transmission channel and user data rate. It was found that the use of adaptive modulation,
and adaptive user allocation reduced the required transmitter power by 5-10dB. Most OFDM systems use a fixed modulation scheme over all subcarriers for simplicity. However each subcarrier in a multiuser OFDM system can potentially have a different modulation scheme depending on the channel conditions. Any coherent or differential, phase or amplitude modulation scheme can be used including BPSK, QPSK, 8-PSK, 16-QAM, 64-QAM, etc, each providing a trade off between spectral efficiency and the bit error rate[4],[5]. The spectral efficiency can be maximized by choosing the highest modulation scheme that will give an acceptable Bit Error Rate (BER). In a multipath radio channel, frequency selective fading can result in large variations in the received power of each subcarrier. For a channel with no direct signal path this variation can be as much as 30 dB in the received power resulting in a similar variation in the SNR. In addition to this, interference from neighbouring cells can cause the SNR to vary significantly over the system bandwidth. To cope with this large variation in SNR over the system subcarriers, it is possible to adaptively allocate the subcarrier modulation scheme, so that the spectral efficiency is maximized while maintaining an acceptable BER.

Using adaptive modulation has a number of key advantages over using static modulation. In systems that use a fixed modulation scheme the subcarrier modulation must be designed to provide an acceptable BER under the worst channel conditions. This results in most systems using BPSK or QPSK. However these modulation schemes give a poor spectral efficiency (1 - 2 b/s/Hz) and result in an excess link margin most of the time. Using adaptive modulation, the remote stations can use a much higher modulation scheme when the radio channel is good. Thus as a remote station approaches the base station the modulation can be increased from 1 b/s/Hz (BPSK) up to 4 - 8 b/s/Hz (16-QAM − 256-QAM), significantly increasing the spectral efficiency of the overall system. Using adaptive modulation can effectively control the BER of the transmission, as subcarriers that have a poor SNR can be allocated a low modulation scheme such as BPSK or none at all, rather than causing large amounts of errors with a fixed modulation scheme. This significantly reduces the need for Forward Error Correction.

IV. RECURSIVE FILTER BASED CHANNEL ESTIMATION ALGORITHMS

The Kalman filter is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms. It estimates the state of a dynamic system. This dynamic system can be disturbed by some noise, mostly assumed as white noise. To improve the estimated state the Kalman filter uses measurements that are related to the state but disturbed as well. Thus the Kalman filter consists of two steps: Prediction and correction [6]. In the first step the state is predicted with the dynamic model. In the second step it is corrected with the observation model, so that the error covariance of the estimator is minimized. This procedure is repeated for each time step with the state of the previous time step as initial value. Therefore the Kalman filter is called a recursive filter. The basic components of the Kalman filter are the state vector, the dynamic model and the observation model [12]-[16].

A. State Space Model for OFDM

Replacing the parameters \( \Delta t, \Delta f \) in with the parameters \( \Delta t=T, \Delta f=1/(NT_s) \) in OFDM systems, we obtain the correlation of the OFDM system channel gain \( H_k[n] \) as

\[
r_k[m] = E\{H_k[n]H_k^*[n-m]\}
= J_0(2\alpha f_mT) \left( \frac{1 - j2\pi(l-k)\tau_{\max}(lNT_s)}{1 + (2\pi(l-k)\tau_{\max}(lNT_s))^2} \right)
\]

(7)

where \( T \) is the OFDM symbol duration, \( 1/NT_s \) is the sampling rate. With the channel correlation, we can apply auto-regressive (AR) model to model the channel \( H_k[n] \).

Define \( h[n] = [H_k[n],...,H_k[n]]^T \). A \( p \)th order AR model for \( h[n] \) is presented as

\[
h[n] = - \sum_{i=1}^{p} A[i]h[n-i] + Q[n]
\]

(8)

Where \( A[1],...,A[p] \) and \( Q \) are \( N \times N \) matrices and \( v[n] \) is the driving noise of AR process, which is an \( N \times 1 \) vector white Gaussian process. \( A[1], ..., A[p] \) and \( Q \) are the model parameters which are obtained. Based on the AR model of the channel, a state space model for OFDM system can be built. Define

\[
x[t] = [h[t]^T,...,h[t-p+1]^T]^T
\]

(9)

we obtain the state equation

\[
x[t] = C[x[t-1]] + G[v[t]
\]

(10)

Where the matrix \( C \) and \( G \) are defined as
The optimal linear estimate of the channel feedback mode. The vector Kalman-filter algorithm gives the optimal linear estimate of the channel [9], [10], [11]. Its drawback is its high complexity. Considering that the dimension of the state vector is \( pN \), which can be significantly high when the number of sub-carriers is large.

\[ h_t = [I_N, 0_N, ..., 0_N] \hat{\mu}_t \]  \hspace{1cm} (20)

Note that the algorithm needs the information symbols \( s_{kt} \)'s so it is working in the training or decision-feedback mode. The vector Kalman-filter algorithm gives the optimal linear estimate of the channel [9], [10], [11]. Its drawback is its high complexity. Considering that the dimension of the state vector is \( pN \), which can be significantly high when the number of sub-carriers is large.

\[ R = E[\text{diag}(s_{kt}^* \text{diag}(s_{kt}))] \]  \hspace{1cm} (21)

The autocorrelation matrix noise, given

\[ Z_t = \text{diag}(s_{kt}^*) \text{diag}(s_{kt}) - R \]  \hspace{1cm} (22)

The gradient noise, given by

\[ \eta_t = Z_t E(h_t - \hat{h}_{t-1}) + \text{diag}(s_{kt}) v_t \]  \hspace{1cm} (23)

and the alternative measurement signal \( f_t \), of dimension \( p \),

\[ f_t = R h_{t-1} + \text{diag}(s_{kt}) \alpha_t = R h_t + \eta_t \]  \hspace{1cm} (24)

We may now instead consider the linear time-invariant model

\[ x_{kt} = A x_t + w_t \]  \hspace{1cm} (25)

\[ f_t = R h_t + \eta_t = R \text{diag}(H) x_t + \eta_t \]  \hspace{1cm} (26)

where \( \eta_t \) acts as measurement noise. The steady-state Kalman state estimator for the model (25),(26) constitutes the General Constant Gain (GCG) estimator for the original model \( h_t = \text{diag}(H) x_t \). The covariance matrix of the gradient noise \( \eta_t \) is required in that calculation [21].

V. SIMULATION PARAMETERS AND VARIABLES

A. OFDM Symbol Parameters

The OFDM parameters utilized in the simulation are listed in Table 1.
TABLE 1. OFDM SYMBOL PARAMETERS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Bandwidth</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Number of Sub-Carriers</td>
<td>128</td>
</tr>
<tr>
<td>Sub-Carrier Separation</td>
<td>10 kHz</td>
</tr>
<tr>
<td>OFDM Symbol Period</td>
<td>111 μs</td>
</tr>
<tr>
<td>Number of Cyclic Prefix</td>
<td>32</td>
</tr>
<tr>
<td>Cyclic Prefix Length</td>
<td>11 μs</td>
</tr>
</tbody>
</table>

B. Simulation Steps

The simulation has been implemented in the following steps:

- Load system parameters
- Generate a random PDP or import a PDP from the SFN simulation
- Initialize and start SNR loop iterations
  - Realize SNR value
  - Initialize and start channel iterations
- Generate Rayleigh channel
- Formulate ISI and ICI components based on channel taps
- Initialize and start ISI-removal iterations (DFE feed-back)
- Generate AWG noise based on SNR value
- Replace previous symbol with next estimated previous symbol
- Generate new current symbol
- Implement channel convolution on actual transmitted symbols (previous and current) and add noise.
- Extract and store symbol estimates from the received signal (Conventional Receiver)
- Calculate and store BER values against respective SNR values
- Plot the BER vs. SNR curves for each case of the extracted symbols
- Calculate the BER values against different QAM Modulations
  - Plot the BER vs SNR curves for each type
- Calculate the MMSE values against the load parameters

VI. RESULTS AND ANALYSIS

Figure 4. SNR vs SER for NMSE Estimator

Figure 4. shows the NMSE Estimator values for 4-QAM as the SNR increases the Symbol Error Rate also increases with higher order QAM. For 4-QAM the SER for a specific SNR is better than all other higher order QAM. If SNR is 20 dB then SER [dB] for 4-QAM is close to 10⁰ for other higher order QAM it is up to 10³.

Figure 5. Prediction NMSE as a function of wavelength

The Mean Square Error for a given wavelengths are shown in the figure 5. The mean square error is minimum for 4-QAM.
Figure 6. SNR vs SER for Kalman/GCG Estimator

Figure 6 shows the performance of Kalman with the GCG channel estimator values. Kalman and GCG estimator performance values are close to each other.

Figure 7. SER vs $E_b/N_0$ of QAM

Symbol Error Rates vs $E_b/N_0$ of QAM is shown in Figure 7 for 4QAM signals.

Figure 8. BER vs SNR for 4-QAM

Figure 9. QAM Constellation

Figure 10. Transmitted Data signal pattern for 128 data bits with random amplitude values

Figure 11. Received data signal pattern for a 128 data bits with amplitude values
VII. CONCLUSION

The purpose of this paper was to give some insight into the power of the OFDM transmission scheme. It has discussed not only the transmission scheme itself, but also some of the problems that are presented in mobile communications as well as the techniques to correct them. Digital Communications is a rapidly growing industry and Orthogonal Frequency Division Multiplexing is on the forefront of this technology. OFDM will prove to revolutionize mobile communications by allowing it to be more reliable and robust while maintaining the high data rate that digital communications demands.

Kalman-filter based channel estimation algorithms are proposed for OFDM systems in a time-varying frequency-selective environment. Kalman filter solution works with training symbols or works in decision-feedback mode, and mixture Kalman filter solution is a blind solution. Though the per-subcarrier Kalman filter with MMSE combiner algorithm is a two-step solution: filtering in time and frequency domain successively, the performance of it is comparable to the much more complicated vector Kalman estimator. The reason behind this may be that the time and frequency components of the Jakes’ model is separable in nature. This paper can be extended on the reduction of performance complexity of the kalman filter and also by using the GCG approximation at different kalman filter taps we can increase the system performance and reduce the complexity.

REFERENCES
Biography

**K. Sreedhar** received the B.Tech. degree in Electronics and Communication Engineering from JNTUH University, Hyderabad, India in 2005 and M.Tech degree in Communication Systems from JNTUH University, Hyderabad, India in 2009. He attended the International Conference on Technology and Innovation at Chennai. He also attended the National Conference at Coimbatore, Tamilnadu, India on INNOVATIVE IN WIRELESS TECHNOLOGY. He is currently working as an Assistant professor in Electronics and Communication Engineering department in VITS (N9) Karimnagar, Andhra Pradesh, India. He has a Life Membership in ISTE. He published Six International papers.

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