



# Connectivity Preserving Distributed Maximizing Coverage Algorithm for Three Dimensional Mobile Sensor Networks

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**ABSTRACT**— *Considering an under supervised 3D space where a group of mobile devices with limited sensing and communicating capabilities are deployed, this paper aims at proposing a decentralized self-deployment algorithm for agents to get maximum connected coverage topology. The problem is modeled as maximization which is solved completely distributed. In fact each agent tries to maximize its sensing volume while preserving connectivity and having high quality communication links. Then each agent knowing distance of its neighbors defines an error function and tries to decrease the error by changing its positions through gradient descent algorithm.*

**Keywords**— mobile sensor networks, coverage, connectivity, QOS, gradient descent

## I. INTRODUCTION

Exploiting mobility of nodes is one of popular ways of providing a better Quality of Service (QOS) in Wireless Sensor Networks. Mobile Sensor Networks (MSNs) can be used in numerous applications such as wildlife habitat monitoring, security surveillance, military and battle-fields, search and rescue, space exploration, target tracking and etc. Size, capabilities and responsibility of nodes depend on application. For instance nodes can vary from very small robomotes [1] to powerful wheeled robots or even unmanned aerial vehicles with complicated reconnaissance equipment.

In most applications, it is very important to have updated and correct information about the environment that magnifies the role of coverage. Furthermore, the nodes need to collaborate and coordinate in many situations, so having a connected network is also mandatory.

Considering importance of coverage and connectivity in MSNs, a distributed connectivity preserving coverage control scheme is proposed in this paper. The network consists of mobile agents that are deployed to perform collaborative monitoring tasks over a given volume. The problem is formulated as coverage maximization subject to connectivity constraint. Considering a desired distance between neighboring nodes, an error function which depends on pairwise distance between nodes is described. The maximization is encoded to an error minimization problem which is solved using gradient descent algorithm and will yield in moving sensors into appropriate positions.

The reminder of this paper is organized as follows: Some related work stated in next section. The considered system model and the problem are described in Section III. Section IV discusses and section V analyses the proposed approach.

Simulation results and performance evaluation are discussed in section VI and section VII concludes the paper.

## II. RELATED WORK

The coverage problem has been extensively studied in the literature in context of multi-robot networks, mobile sensor networks and wireless sensor and actor networks. In most of papers, the network is considered in 2D space where mobile node deployed randomly and can move around to adjust their positions after initial deployment in order to reduce their coverage overlaps and maximize area coverage while preserving connectivity. Many approaches presented reducing the 3D geometry to the 2D [2]–[6]. Different modeling and strategies such as virtual forces, optimization and predefined pattern was proposed. In [7] Megerian and Potkonjak model the Coverage Problem in Wireless Sensor Networks as Integer Linear Programming (ILP) and solve the problem using a greedy algorithm. Similarly, Nakamura et al [8] used ILP modeling and solve it with the commercial optimization package CPLEX [9]. Yiannis and Tzes in [10] and [11] modeled the coverage problem as a constrained maximization. The proposed algorithm in this paper is mainly inspired from the approaches used to model the problem in [10] and [11]. In one of the other group of coverage improvement algorithms such as [12], pattern-based strategies are presented in 2D and 3D space. In such algorithms target locations that can provide both coverage and network connectivity requirements are computed based on a predefined pattern and mobile nodes will deploy in the target positions. Alam and Haas [13] proposed a deployment pattern that generates the Voronoi tessellation of truncated octahedron in 3D space. Achieving full-coverage and k-connectivity in three dimensional spaces, some lattice was proposed in [14] and [15].

Wu et al [16] used Delaunay Triangulation to solve the coverage problem and finding the position of sensors in the environment. Vieira et al [17] use computational geometry and graph theory to find a solution. In virtual force-based node movement strategies [18]-[20], mobile nodes affect each-others. In detailed form, two mobile nodes expel each other if their distance is close than a threshold or attract each other if their distance is too far away. Other types of forces such as attractive force of boundaries and the repulsive forces of obstacles also can be considered. The total force is vector addition of all forces which specifies the direction and distance of mobile node movement. In [21] and [22], the coverage problem is modeled as an optimization and solved through evolutionary algorithms PSO and ant colony optimization respectively.

### III. PROBLEM FORMULATION

In first part of this section the system is described via determining its module features. Then the problem is formulated as a constrained maximization in.

#### A. Main Assumption

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Let assume that network includes set of mobile homogenous agents denoted by  $A = \{a_1, a_2, \dots, a_n\}$ . Nodes are distributed randomly in 3D space called volume of interest that we represent by  $\mathcal{V} \subset \mathbb{R}^3$ . Let assume that all nodes are location aware and each node knows its position  $p_i$  using a GPS or other node localization approaches and is aware of positions of its one-hop neighbors. All agents equipped with wireless transceiver and the transmission range of all agents is equal and is denoted by  $R_c$  and is much less than dimension of volume of interest. The communication or transmission range of a node is defined as maximum Euclidean distance that its radio signals effectively can reach to other agents. Considering communication range of each agent, communication volume of agent  $i$  is denoted by  $c_i$  and is a sphere centered in agent position i.e.

$$c_i = \{p \in \mathbb{R}^3 \mid \|p - p_i\| \leq R_c\}, i \in I_n \quad (1)$$

Similarly, the sensing volume of agent  $i$  represented by  $s_i$  is a sphere of radius  $R_s$ , having the sensor node at its center i.e.

$$s_i = \{p \in \mathbb{R}^3 \mid \|p - p_i\| \leq R_s\}, i \in I_n \quad (2)$$

where  $\|\cdot\|$  stands for standard Euclidean norm and  $I_n$  is defined according to following equation:

$$I_a = \{i \in \mathbb{N} \mid i \leq a\}, a \in \mathbb{N}. \quad (3)$$

Also it is assumed sensing and transmission ranges of agents are identical but transmission range is greater than sensing radius. Let the density function  $\phi: \mathcal{V} \subset \mathbb{R}^3 \rightarrow \mathbb{R}_+$  demonstrate probability of occurring an event in an arbitrary point  $p \in \mathcal{V}$  in other word the function explain importance of each point. . Another important function is  $f_i: \mathcal{V} \subset \mathbb{R}^3 \rightarrow \mathbb{R}_+$  which is determined by following equation:

$$f_i(p) = \begin{cases} 1 & \text{if } p \in s_i \\ 0 & \text{if } p \notin s_i \end{cases} \quad (4)$$

Considering aforementioned functions and notations the main objective of this paper could be expressed as proposing a distributed algorithm to maximize following function  $\mathcal{C}$ :

$$\mathcal{C} = \bigcup_{i=1}^n \int_{\mathcal{V}} f_i(p) \phi(p) dp, i \in I_n \quad (5)$$

If the position matrix considered as  $P = (p_1, p_2, \dots, p_n)$  the maximization can be defined as following constrained optimization:

$$\begin{aligned} & \text{find } P: \\ & \text{maximize } \mathcal{C} \\ & \text{subject to: not losing connectivity} \end{aligned} \quad (6)$$

#### B. Voronoi Diagram

The Voronoi diagram for a set of points in a given space  $\mathbb{R}^d$  is the partitioning of that space into regions such that all locations within any one region are closer to the generating point than to any other. In three dimensions, a Voronoi cell is a convex polyhedron formed by convex faces, as shown in the Fig. 1. The  $i$ th node partition or  $i$ th node Voronoi cell is defined as

$$v_i = \{p \in \mathcal{V} \mid \|p - p_i\| \leq \|p - p_j\|, \forall j \in I_n\} \quad (7)$$

Using similar notation introduced in [18] and [19] for 2D space, the limited Voronoi cell of  $i$ th node is defined by equation (8) and is depicted Fig. 1.

$$v_i^l = v_i \cap s_i, i \in A_n \quad (8)$$

Demonstration of Voronoi Cell Due to lack of considering sensing and region overlaps, the equation (5) could be modified to equation (9).

$$\mathcal{C} = \bigcup_{i=1}^n \int_{v_i} f_i(p) \phi(p) dp \quad (9)$$

Through limited Voronoi cell definition equation (10) can be obtained

$$v_i^l = \int_{v_i} f_i(x) \phi(x) dx \quad (10)$$

and by substituting equation(10) into equation(9) the result would be:

$$c = \bigcup_{i=1}^n v_i^l \quad (11)$$

So the union of limited Voronoi cells could be considered equal to total coverage of network and as result optimization defined by equation (6) is revised in the following form

find P:

$$\text{maximize } \bigcup_{i=1}^n v_i^l \quad (12)$$

subject to: not losing connectivity

Denoting  $N_i$  for one-hop neighbor of  $i$ th agent the set  $E_i$  is defined as:

$$E_i = \{(i, j) | j \in N_i\}, \quad i, j \in I_n \quad (13)$$

Exploiting divide and conquer strategy equation (12) could be rewritten in the following form

find  $p_i$ :

$$\text{maximize } v_i^l$$

subject to:  $E_i \neq \emptyset$  (14)

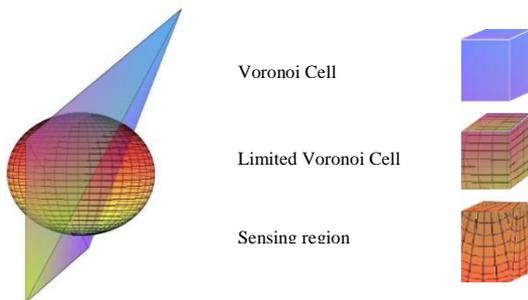


Fig. 1 Demonstration of limited Voronoi cell

In fact the maximization problem defined by equation (14) is divided into sub-problems which are solvable through local information and in a distributed manner.

#### IV. PROPOSED ALGORITHM

Solving sub-problems in last section means increasing volume of Voronoi cell of each agent without losing connectivity of the agent with the others, so in this section a parallel constrained maximizing algorithm running in all agents is proposed.

To perform the algorithm, first desired distance between two neighbors ( $\alpha$ ) is specified. The value of  $\alpha$  would determine importance of coverage and connectivity, such that by increasing or decreasing  $\alpha$  coverage or connectivity is made more important in other word having a greater  $\alpha$  means increasing coverage of each node and thus increasing total coverage of network and decreasing  $\alpha$  means having lower distance between nodes, so having a more compact and connected network. As transmission is one of energy-consuming act of network, it should also be noted that increasing or decreasing  $\alpha$  will affect the energy consumption of network since decreasing distance of two neighbor nodes means increasing signal power and decreasing node distance means decreasing power used for transmission.

In second step by determining  $\alpha$ , an error function easily could be obtained. The error functions assign an error to each distance. In fact knowing the distance between nodes  $i$  and  $j$  represented by  $d(i, j)$ , the function assigns an error to  $d(i, j)$ . The amount of error is zero when  $d(i, j)$  equals  $\alpha$  and will increase as the distance become more or less. The error function is plotted in Fig. 2 ( $\alpha = 9.5, a = 5, b = 1$ ).

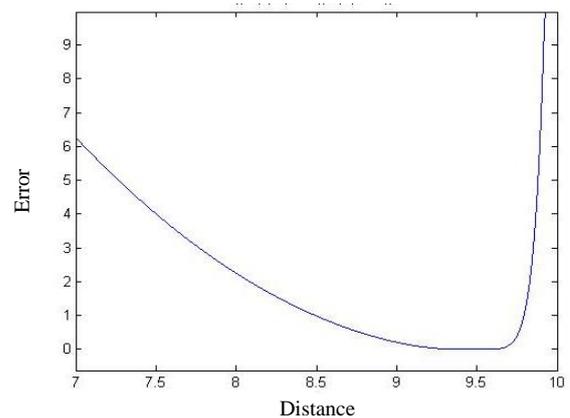


Fig. 2 Error Function

Denoting position of  $i$ th node or  $i$ th column of  $P$  as  $p_i$ , obtaining the following equations seems straightforward.

$$p_i(x_i, y_i, z_i) \in \mathbb{R}^{2n} \quad (15)$$

$$\Delta p_{ij} = p_i - p_j \quad (16)$$

$$d_{ij} = \sqrt{\Delta p_{ij}^T \Delta p_{ij}} \quad (17)$$

The error between nodes  $i$  and  $j$  depends on difference of their distance from  $\alpha$  through the following equation:

$$e_{ij} = a \times (\exp(b \times (d_{ij} - \alpha)) - 1) (d_{ij} - \alpha) \quad (18)$$

So the total error of  $i$ th node is

$$E_i = \frac{1}{|N_i|} \sum_{j \in N_i} e_{ij}^2 \quad (19)$$

Position of  $i$ th node is changed through equation (20)

$$p_{i(t+1)} = p_{i(t)} - \eta \frac{\partial E_i}{\partial p_i} \quad (20)$$

To getting derivate from  $E_i$  we have

$$\frac{\partial E_i}{\partial p_i} = \frac{1}{|N_i|} \sum_{j \in N_i} 2 e_{ij} \frac{\partial e_{ij}}{\partial p_i} \quad (21)$$

$$\frac{\partial E_i}{\partial p_i} = \frac{1}{|N_i|} \sum_{j \in N_i} 2 e_{ij} \frac{\partial e_{ij}}{\partial d_{ij}} \frac{\partial d_{ij}}{\partial p_i} \quad (22)$$

$$\frac{\partial E_i}{\partial p_i} = \frac{1}{|N_i|} \sum_{j \in N_i} 2 e_{ij} \frac{\partial e_{ij}}{\partial d_{ij}} \frac{\partial d_{ij}}{\partial \Delta p_{ij}} \frac{\partial \Delta p_{ij}}{\partial p_i} \quad (23)$$

$$\frac{\partial d_{ij}}{\partial \Delta p_{ij}} = \frac{\Delta p_{ij}}{d_{ij}} \quad (24)$$

$$\frac{\partial \Delta p_{ij}}{\partial p_i} = 1 \quad (25)$$

$$a \times (\exp(b \times (d_{ij} - \alpha)) \times (b \times (d_{ij} - \alpha) + 1) - 1) \frac{\partial e_{ij}}{\partial d_{ij}} \quad (26)$$

Using the above equations we get

$$\frac{\partial E_i}{\partial p_i} = \frac{1}{|N_i|} \sum_{j \in N_i} 2 e_{ij} \frac{\Delta p_{ij}}{d_{ij}} (\exp(a \times (d_{ij} - \alpha)) \times (a \times (d_{ij} - \alpha) + 1) - 1) \quad (27)$$

By substituting equation (27) to equation (20), the following equation is obtained:

$$p_{i(t+1)} = p_{i(t)} - \frac{2\eta}{|N_i|} \sum_{j \in N_i} e_{ij} \frac{\Delta p_{ij}}{d_{ij}} \times a \times (\exp(b \times (d_{ij} - \alpha)) \times (b \times (d_{ij} - \alpha) + 1) - 1) \quad (28)$$

In fact agents, using equation (28) change their positions in each time step such that  $C$  is a non-decreasing function of time. The modification will halt when

$$p_{i(t)} - p_{i(t+1)} < \epsilon \quad (29)$$

## V. ALGORITHM ANALYSIS

In this section the algorithm is investigated. The algorithm will be verified in maximizing coverage and not losing connectivity. If the minimum distance between two neighbors is set to  $\alpha$ , the volume of limited Voronoi cell of  $i$ th node can be estimated by a sphere with radius equal to  $\frac{\alpha}{2}$  so the result would be inequality of equation (30)

$$v_i^1 \geq \frac{1}{6} \pi \alpha^3 \quad (30)$$

Considering equation (30) and (11) the following equations will tie  $\alpha$  in total coverage of network.

$$C \geq \bigcup_{i=1}^n \frac{1}{6} \pi \alpha^3 \quad (31)$$

$$C \geq \frac{n}{6} \pi \alpha^3 \quad (32)$$

So by increasing  $\alpha$ , total coverage of network will be increased and vice versa. Usual exponential model that is used for modeling communication in wireless systems is plotted in Fig. 3 for parameter  $\sigma^2 = 0.5$ . The vertical column represent remaining portion of signal power versus distance from the node. Considering this model it is completely clear that lower distance between two nodes means higher connectivity or better quality of connections and being more far away result in lower connectivity. So by adjusting  $\alpha$  desired level of connectivity can be gained. It is worth nothing to say that the desired value of  $\alpha$ , surely is lower than communication range to prevent links from failing.

Another problem is whether the algorithm can find the optimum position or not. By noticing the convexity of error functions and lack of any local minima, the gradient descent

algorithm will guarantee to find a position in which global minimum error occurred. By following discussion we can conclude by setting  $\alpha$  lower than  $R_c$ , the network will stay connected and by considering  $\alpha$  near sensing radius the sensing volumes are maximized. The exact value of  $\alpha$  is determined in simulation section.

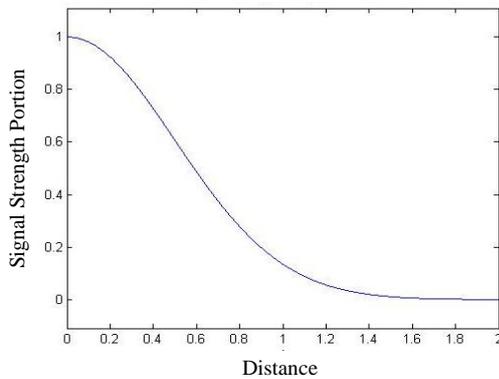


Fig. 3 Exponential model of connection

### VI. SIMULATION AND RESULTS

In this section some simulations will be shown to test the effectiveness and efficiency of the proposed algorithm. The network includes 12 agents deployed randomly on the region.  $\mathcal{V} \subset \mathbb{R}^3$  During simulation the covered volume is increasing until it reaches the maximum possible portion of environment. The simulation parameters are summarized in table 1.

TABLE I  
SIMULATION PARAMETERS

NODE PLACEMENT	RANDOM
No of agents	12
Communication Range	10
Sensing Range	5
<b>a</b>	1
<b>b</b>	5
<b><math>\alpha</math></b>	9.5
<b><math>\eta</math></b>	0.2
<b><math>\epsilon</math></b>	1

Considering the abovementioned parameters, the agent's initial, intermediate and final positions are shown in Fig. 4. Due to having desired distance near sensing radii, as can be seen in final state of the network, the agents self-organize themselves in a way that there is minimum overlapping between sensing volumes and total coverage is of network is some value near coverage upper bound i.e.  $\mathcal{C} = \frac{n}{6} \pi R_s^3$ .

For better understanding, top views of all 3D pictures are shown. As it is seen the initial state of the network is so compact and sensing volume have overlaps but in the final

state the surveyed volume by the network is maximized without losing connectivity of agents.

In fact, by adjusting  $\alpha$  the importance of coverage and connectivity can be adjusted such that in this case coverage is much more important than connectivity. It is obvious that network parameters could be adjusted in a way that agents opt to preserve connectivity in comparison with maximizing coverage. The sensing radii is considered half of communication radii which means the agents with tangent sensing volumes are connected which are connected through red lines.

Comparing this work to previous ones, this algorithm tries to maximize the coverage completely distributed and in a self-organizing manner. In simulation process making the simulation more realistic agents are selected randomly to update their positions and the selected agent, updated its position with probability of %95 and it can be mentioned that all simulations converged to optimum solutions in below ten iterations. In short term the solution for constrained maximization is obtained is step by step and in a monotonic manner.

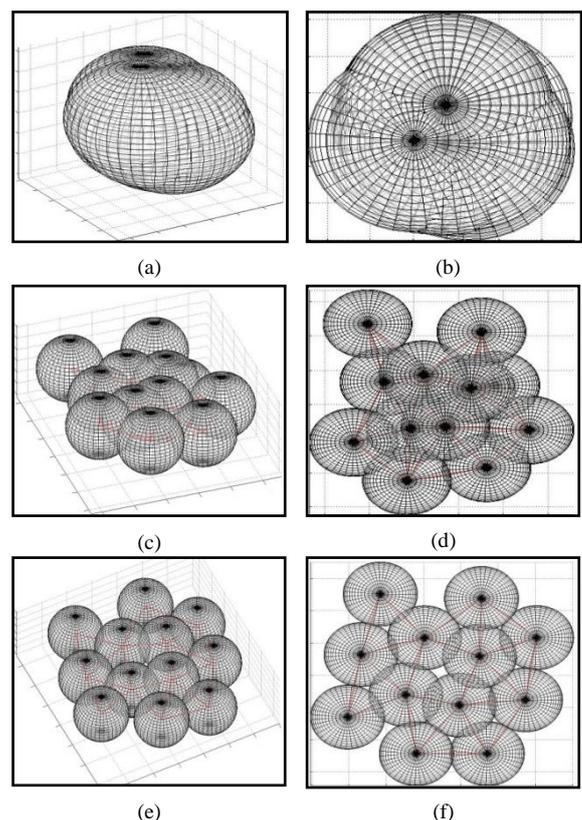


Fig. 4 Exponential model of connection (a) Lateral 3D view of initial deployment (b) Top view of initial deployment (c) lateral 3D view of agents deployment after three iteration (d) Top view of agents deployment after three iteration (e) lateral 3D view of final deployment ( after ten iteration) (f) Top view of final deployment ( after ten iteration).

## VII. CONCLUSION

In this paper a motion coordination algorithm for mobile agents deployed in a 3D wireless sensor network is proposed. It is assumed that agents have sphered sensing and communication region. The total coverage of network and quality of links can be controlled through parameter  $\alpha$  and as seen in simulations by adjusting  $\alpha$  near sensing radii the approach would lead the network to a configuration with maximum connected coverage. Simulation results show the efficiency and fast convergence of algorithm.

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